

Poisson Distribution

How to recognize:

1. n is large
2. p is small and remains the same
3. Usually the events occur over an interval like time, area, or volume.

i.e. Errors per second in data transmission, plants per acre, fecal coliforms per gallon.

The formula for the Poisson Distribution

The probability of X occurrences in an interval, where λ is the mean number of occurrences per unit, is:

$$P(X; \lambda) = \frac{e^{-\lambda} \lambda^X}{X!}$$

Round your answers to four decimal places!

e.g.1

If there are 200 typographical errors randomly distributed in a 500-page manuscript, find the probability that a given page contains exactly three errors.

Because there are 200 errors per 500 pages

$$\lambda = 200 / 500 = 0.4$$

$$\text{and } X = 3$$

$$P(3;0.4) = \frac{e^{-0.4}(0.4)^3}{3!}$$

$$P(3;0.4) = 0.0072$$

e.g.2

A video tape has an average of one defect every 1000 feet. Find the probability of 5 defects in 3000 feet.

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A video tape has an average of one defect every 1000 feet. Find the probability of 5 defects in 3000 feet.

$$\lambda = 0.001 * 3000 = 3$$

$$X = 5$$

$$P(5;3) = \frac{e^{-3}(3)^5}{5!}$$

$$P(5;3) = 0.1008$$

e.g.3

A telephone tech-support call center gets about 3 calls per hour on its 800 number. For a randomly selected hour find these probabilities.

1. The call center will receive zero calls.
2. It will receive 1 call.
3. It will receive 2 calls.
4. It will receive 3 or more calls calls.

e.g.3 Answer for 1.

A telephone tech-support call center gets about 3 calls per hour on its 800 number. For a randomly selected hour find these probabilities.

1. The call center will receive zero calls.

$$P(0;3) = \frac{e^{-3}(3)^0}{0!} = 0.0498$$

e.g.3 Answer for 2.

A telephone tech-support call center gets about 3 calls per hour on its 800 number. For a randomly selected hour find these probabilities.

2. It will receive 1 call.

$$P(1;3) = \frac{e^{-3}(3)^1}{1!} = 0.1493$$

e.g.3 Answer for 3.

A telephone tech-support call center gets about 3 calls per hour on its 800 number. For a randomly selected hour find these probabilities.

3. It will receive 2 calls.

$$P(2;3) = \frac{e^{-3}(3)^2}{2!} = 0.2240$$

e.g.3 Answer 4.

A telephone tech-support call center gets about 3 calls per hour on its 800 number. For a randomly selected hour find these probabilities.

4. It will receive 3 or more calls.

Probability of 3 or more = 1 - probability of at most two.

$$\begin{aligned}\text{Probability of at most 2} &= P(0;3) + P(1;3) + P(2;3) \\ &= 0.0498 + 0.1493 + 0.2240 = 0.4231\end{aligned}$$

$$\text{Prob of 3 or more} = 1 - 0.4231 = 0.5769$$

