

Multinomial Distribution

How to recognize:

1. More than two outcomes possible.
2. Fixed or known number of trials.
3. Probability of success remains the same for all trials.

Formula for the Multinomial Distribution

$$P(x) = \frac{n!}{X_1! * X_2! * \dots * X_n!} * P_1^{X_1} * P_2^{X_2} * \dots * P_n^{X_n}$$

n = number of trials, and
if E1, E2, ... En are all the possible outcomes, then
P1, P2, ... Pn are the probabilities of each E happening
X1, X2, ... Xn are the number of times each E happens

note:

$$X_1 + X_2 + \dots + X_n = n$$

and

$$P_1 + P_2 + \dots + P_n = 1$$

e.g. 1

In a large city 50% of the people choose a movie, 30% choose dinner, and 20% choose shopping as their favorite leisure activity.

If 5 people are selected at random, find the probability that three are going to a movie, one to dinner, and one is going shopping.

$$n = 5$$

$$X_1 = 3 \quad X_2 = 1 \quad X_3 = 1$$

$$P_1 = 0.5 \quad P_2 = 0.3 \quad P_3 = 0.2$$

$$P(x) = \frac{5!}{3!1!1!} * 0.5^3 * 0.3^1 * 0.2^1$$

$$P(X) = 0.15$$

e.g. 2

Before an iPod leaves the factory it is given a quality control check. The probability that it has 0, 1, or 2 defects is 0.88, 0.08, and 0.04 respectively.

In a sample of 16 find the probability that 9 will have no defects, 4 will have 1 defect and 3 will have 2 defects.

e.g. 2

Before an iPod leaves the factory it is given a quality control check. The probability that it has 0, 1, or 2 defects is 0.88, 0.08, and 0.04 respectively.

In a sample of 16 find the probability that 9 will have no defects, 4 will have 1 defect and 3 will have 2 defects.

$$n = 16$$

$$X_1 = 9 \quad X_2 = 4 \quad X_3 = 3$$

$$P_1 = 0.88 \quad P_2 = 0.08 \quad P_3 = 0.04$$

$$P(x) = \frac{16!}{9!4!3!} * 0.88^9 * 0.08^4 * 0.04^3$$

$$P(x) = 0.00033 \quad \text{or } 3.3E-4 \quad \text{or } 3.3 * 10^{-4}$$

e.g. 3

The probability that a page of the Cardinal Times will have 0, 1, 2, or 3 typo's are 0.79, 0.12, 0.07, and 0.03 respectively.

If 8 pages are inspected, find the probability that 4 pages will contain no errors, 2 will contain 1 error, 1 will contain 2 errors and 1 will contain 3 errors.

e.g. 3

The probability that a page of the Cardinal Times will have 0, 1, 2, or 3 typo's are 0.79, 0.12, 0.07, and 0.03 respectively.

If 8 pages are inspected, find the probability that 4 pages will contain no errors, 2 will contain 1 error, 1 will contain 2 errors and 1 will contain 3 errors.

$n = 8$

$X_1 = 4 \quad X_2 = 2 \quad X_3 = 1 \quad X_4 = 1$

$P_1 = 0.79 \quad P_2 = 0.12 \quad P_3 = 0.07 \quad P_4 = 0.03$

$$P(x) = \frac{8!}{4!2!1!1!} * 0.79^4 * 0.12^2 * 0.07^1 * 0.03^1$$

$P(x) = 0.0099$

eg 4 Suppose you toss a fair die 12 times. What is the probability that each face value (1-6) will occur exactly twice?

Therefore, the probability of rolling exactly 2 of each face value on a fair die is about 0.35%.

eg 5 A bowl has 2 gold marbles, 3 blue marbles and 5 green marbles. A marble is randomly selected and then placed back in the bowl. You do this 13 times. What is the probability of choosing 3 gold marble, 4 blue marble and 6 green marbles?

According to Mendel's theory, if tall and colorful plants are crossed with short and colorless plants, the probabilities are $\frac{9}{16}$, $\frac{3}{16}$, $\frac{3}{16}$, and $\frac{1}{16}$ for tall and colorful, tall and colorless, short and colorful, and short and colorless, respectively. If 22 plants are selected find the probability that 12 will be tall and colorful, 4 will be tall and colorless, 5 will be short and colorful, and 3 will be short and colorless.

