Questions?

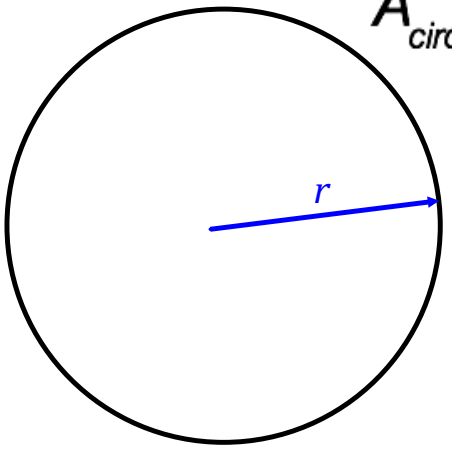
March 30, 2020

Finding area:

Circles, Sectors, Segments, Lunes

Circles

$$A_{circle} = r^2\pi$$

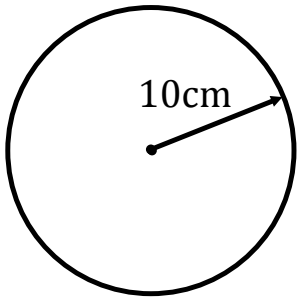


Example

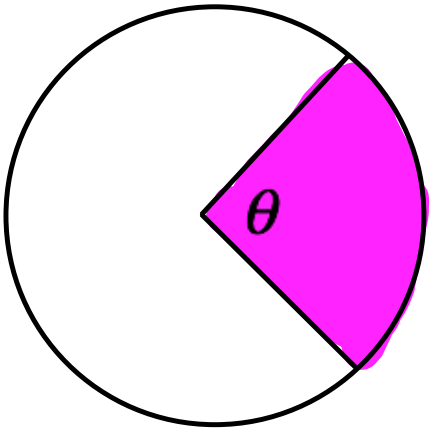
Find the area of a circle with radius 10 cm.

$A_{circle} = 10^2\pi \text{ cm}^2$

$= 100\pi \text{ cm}^2$



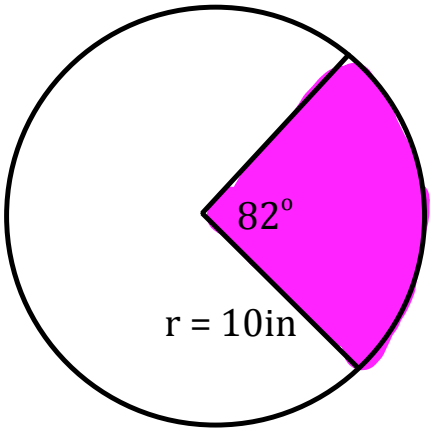
Sectors



$$A_{sector} = \frac{\theta}{360} r^2 \pi$$

Remember: $\theta/360$ is the portion of the circle we are interested in.

Example



$$Area = \frac{82}{360}10^2\pi$$

$$Area = 22.78\pi in^2 \approx 71.56 in^2$$

Remember: 82/360 is the portion of the circle we are interested in.

The Triangle

With the triangle's height (drawn in red) where it is, the radius is both the hypotenuse of the right triangle AND the base of the whole triangle. (note: "*" means multiply)

The height is found by sin(θ) = height/radius.

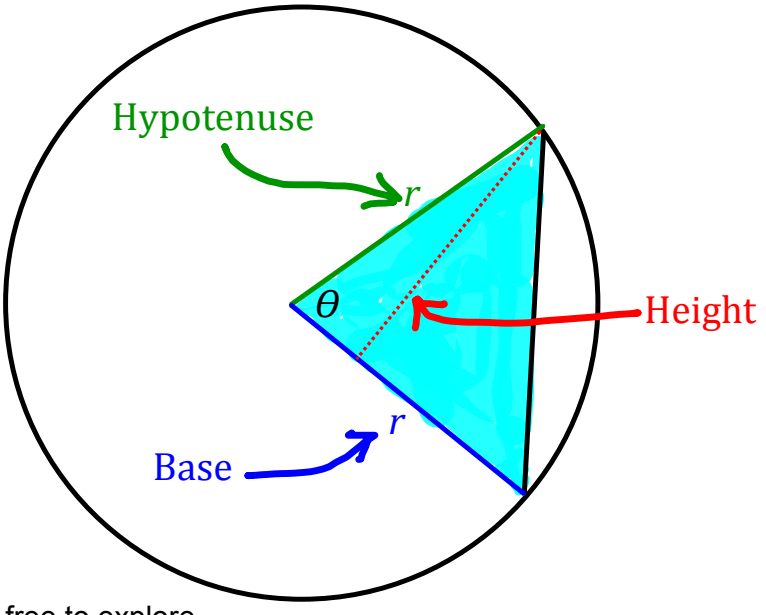
So height = r*sin(θ).

The base of the triangle is also r.

Area = ½*base * height

Area = ½*r * r*sin(θ)

$$Area_{triangle} = \frac{1}{2} r^2 \sin(\theta)$$



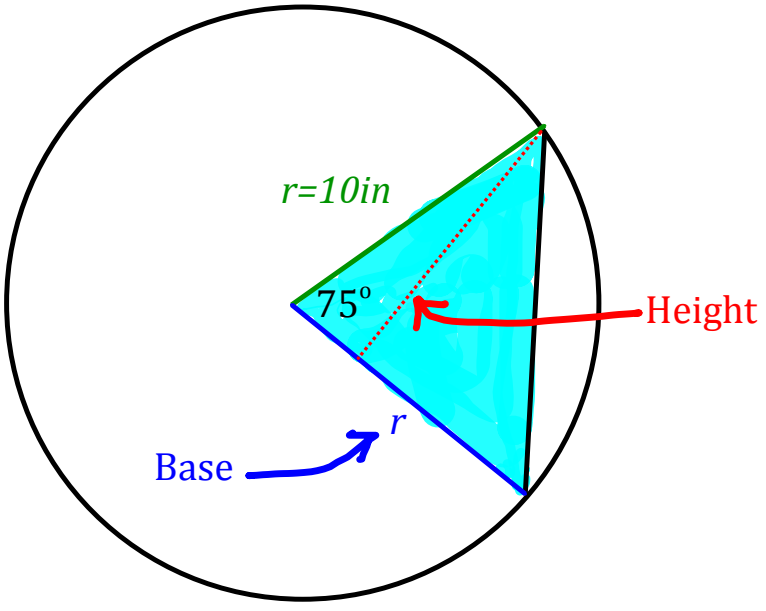
There are other ways to do this. Feel free to explore.

Example

$$Area_{triangle} = \frac{1}{2} r^2 \sin(\theta)$$

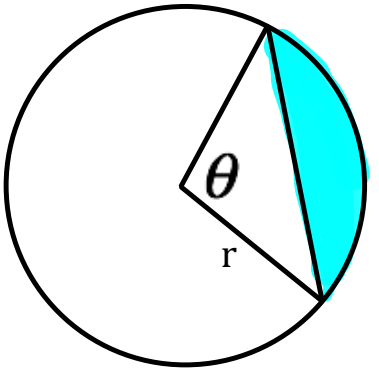
Area = ½ 10^2 sin(75)

≈ 48.3 in^2



Segments

As you probably guessed, the area of the segment is the area of the sector minus the area of the triangle. You can work this out piecemeal.



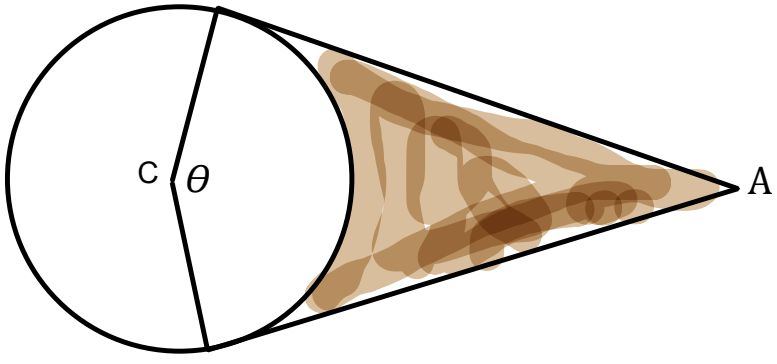
$A_{segment} = A_{sector} - A_{triangle}$

But if you like complex formulæ:

$$Area_{segment} = \frac{\theta}{360} r^2 \pi - \frac{1}{2} r^2 \sin(\theta)$$

Can you work out how to find this area?

It helps to think of a line segment between C and A, and to remember we are working with tangents, and therefore right triangles.



Of course you will need more information, but you could be given the distance of CA or the distance of the tangent segment, or the distance of CA minus the radius.

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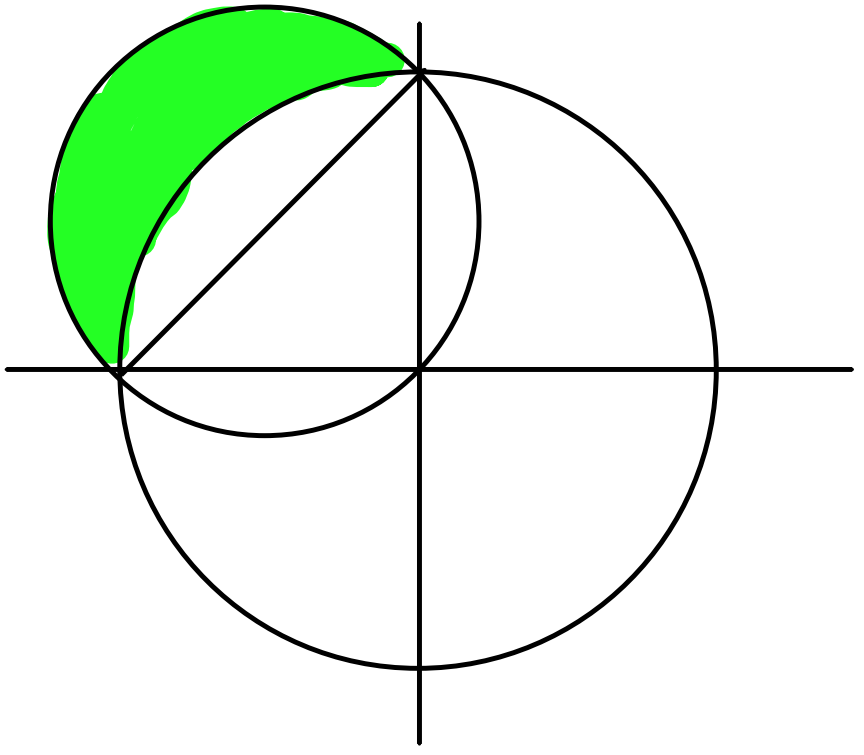
Lunes



This is easier than it looks.

The area of a lune is the area of the large segment minus the area of the small segment.

$$Area_{segment} = \frac{\theta}{360} r^2 \pi - \frac{1}{2} r^2 \sin(\theta)$$

Calculate this for both segments.

Then subtract the smaller area from the larger area.

Some lunes are easier to calculate depending on where the circle centers lie with respect to each other.

Just be careful not to mix the central angles and not to mix the radii.

Example

The big circle:

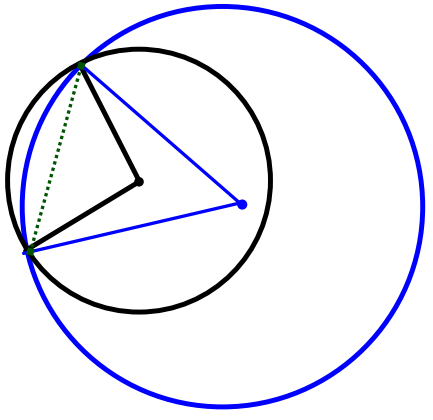
r = 22in

$\theta = 28^o$

$28/360\ 22^2\ \pi - ½\ 22^2 \sin(28)$

$118.27 - 113.61$

$A_{segment} = 4.66\ in^2$



The small circle:

r = 13in

$\theta = 95^o$

$95/360\ 13^2\ \pi - ½\ 13^2 \sin(95)$

$140.11 - 84.18$

$A_{segment} 55.93\ in^2$

Area of the lune = $55.93 - 4.66 = 51.27\ in^2$

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