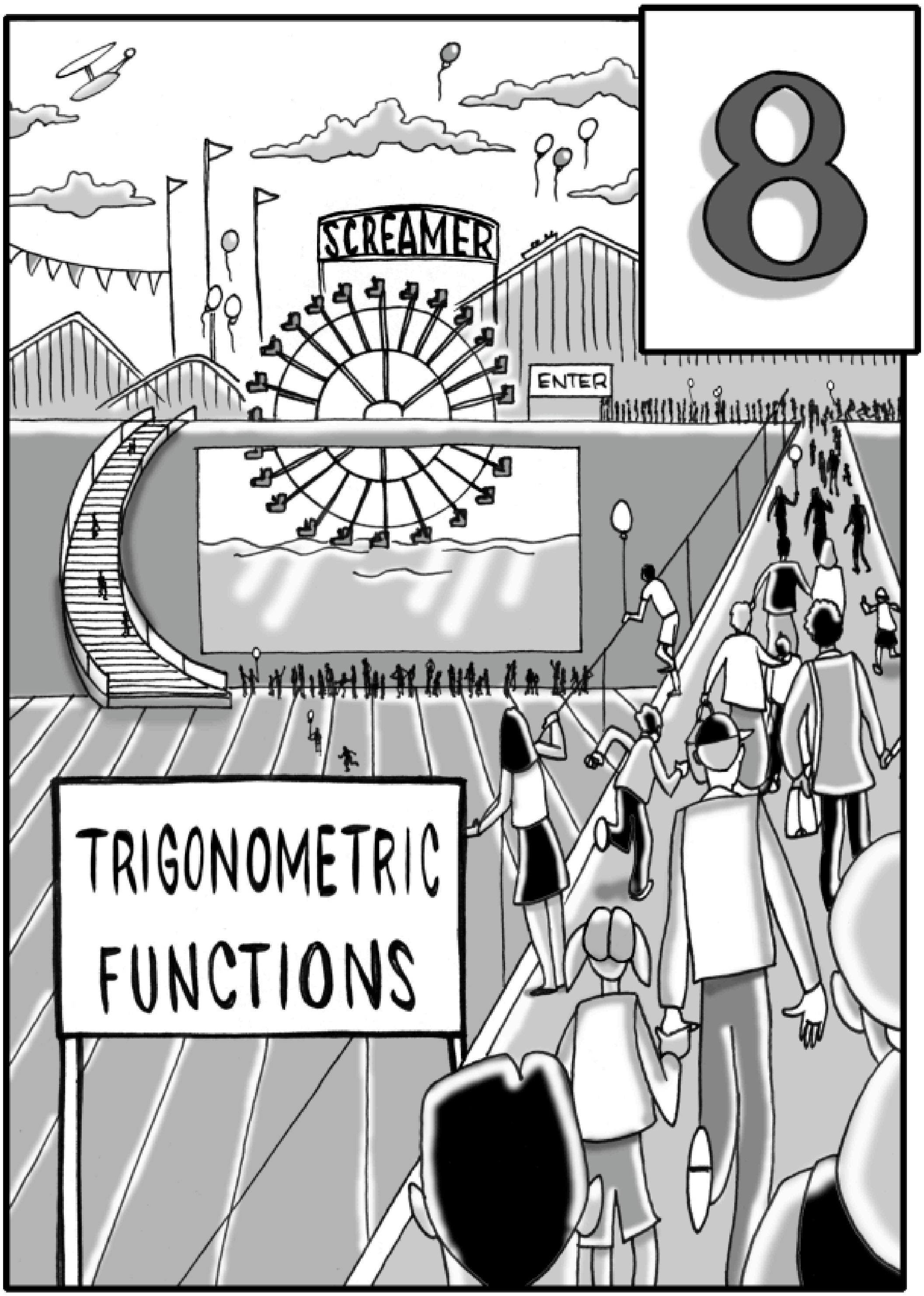

8
SCREAMER
ENTER
TRIGONOMETRIC
FUNCTIONS

Chapter 8 Teacher Guide

Section	Lesson	Days	Lesson Title	Materials	Homework
8.1	8.1.1	1	Introduction to Cyclic Models	• White glue, zipper-seal bags, water, meter sticks, string, butcher paper, binder clips, food coloring, scissors, paper towels • Lesson 8.1.1 Res. Pg. (optional) • Optional: needle or pin	8-3 to 8-11
	8.1.2	2	Graphing the Sine Function	• Cardboard boxes • Box cutter • Colored overhead pens • Lesson 8.1.2A, B, C, and D Res. Pgs. • Toothpicks • Transparencies • Cardstock	8-15 to 8-24 and 8-25 to 8-33
	8.1.3	1	Unit Circle $\leftrightarrow$ Graph	• Lesson 8.1.3 Res. Pg.	8-37 to 8-45
	8.1.4	2	Graphing and Interpreting the Cosine Function	• Lesson 8.1.4A and B Res. Pgs.	8-53 to 8-62 and 8-63 to 8-72
	8.1.5	1	Defining a Radian	• Circular objects • String, Wire or Tape • Scissors	8-78 to 8-86
	8.1.6	1	Building a Unit Circle	• Lesson 8.1.6 Res. Pg.	8-91 to 8-98
	8.1.7	1	The Tangent Function	• Lesson 8.1.7 Res. Pg. • Transparencies • Overhead pens	8-104 to 8-112
8.2	8.2.1	1	Transformations of $y = \sin x$	• Computer and projector • Dynamic wrapping applet	8-116 to 8-122
	8.2.2	1	One More Parameter for a Cyclic Function	None	8-127 to 8-136
	8.2.3	1	Period of a Cyclic Function	• Curves from Lesson 8.1.1 • Meter sticks	8-143 to 8-151
	8.2.4	1	Graph $\leftrightarrow$ Equation	None	8-158 to 8-165
Chapter Closure			Varied Format Options		

Total: **13 days plus optional time for Chapter Closure**

8.1.1 What cyclic relationships can I model?

Introduction to Cyclic Models

In this chapter, you will learn about a new family of functions that are very useful for describing relationships that are **cyclic** (have repeating cycles), like the height of the ocean as the tides fluctuate between high and low or the distance of a swinging pendulum from its center point.

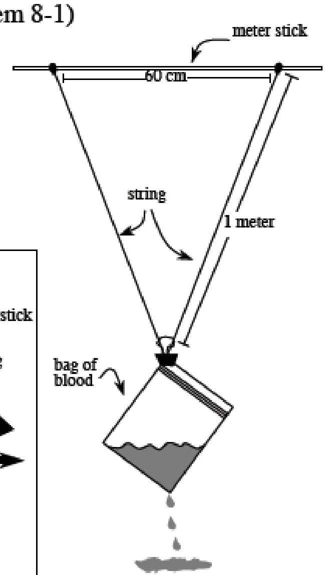
8-1. EMERGENCY!

Nurse Nina rushes through the hospital with one hand on her clipboard and the other pulling a portable blood stand. The bags of blood swinging from the stand are needed in the emergency room (E.R.) ***'stat'*** for three patients in severe need. She is so intent on delivering the blood in time that she does not notice that one of the bags has a small hole in it and is dripping on the floor behind her. As she reaches the E.R., she is horrified to see a small pool of blood beginning to form on the floor. Looking down the corridor from where she came, she sees the trail of blood that dripped and notices that it forms a very interesting pattern.

What shape do you think the trail of blood created?

Your task: Use the materials and instructions provided by your teacher to re-create the pattern that the nurse saw on the hallway floor behind her. Make as many observations as you can about the shape you see and how that shape relates to what was happening to the bag of blood. Be sure to keep track of all the details. Be prepared to explain your observations to the class. (Note: Complete directions for conducting this experiment are on the next page.)

Directions for the Pendulum Experiment (Problem 8-1)

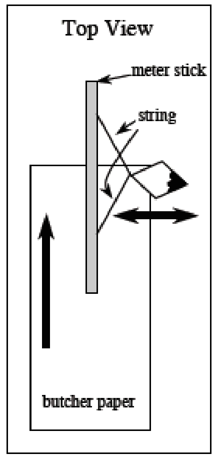


Find a space in the classroom where there is room to work. You may need to move your table or desks out of the way.

Set up your pendulum by attaching your bag of liquid to a meter stick using string, as shown at right.

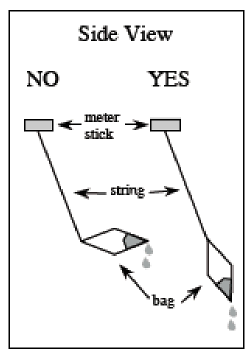
Assign the following tasks:

- One student holds the meter stick from which the pendulum swings.
- One student holds the bag, releases it when ready, and then stops it when it reaches the end of the paper.
- Two students hold the butcher paper, one at each end, and slide it *at a constant rate* underneath the swinging pendulum. Be sure to slide the paper parallel to the meter stick and perpendicular to the swing of the pendulum, as shown in the diagram at right.



Tips for a successful experiment:

- The student holding the meter stick should hold the places where the string attaches. Otherwise, the string has a tendency to slide toward the center of the meter stick.
- Use as much of the string as you can so that your pendulum is as long as possible.
- When you are ready, have your teacher cut a *very* small hole in the corner of the bag. The student holding the bag should pinch this hole closed until the pendulum begins swinging.
- Be sure to slide the butcher paper under the pendulum *at a constant rate*.
- As you pull the bag up to start the pendulum swinging, hold it taut from the bottom corner so that the bag remains in line with the string (see diagram at right).
- Have an extra bag ready to place the dripping bag into when the experiment is complete.



8-2. Now you will make predictions about a new curve.

a. With your team, design your own curve similar to the one already created. Decide exactly how you want it to look. How long do you want each cycle? How tall do you want it to be? Where do you want it to be on your paper? Where do you want it to start? Once you have decided on your new curve, make a sketch of it on your paper.

b. Predict how you could conduct an experiment to get exactly the curve you have described. How fast should you move the paper? Where would you start it? How high would you start your pendulum? Be prepared to share your predictions and their **justifications** with the class.

Review & Preview

8-3. Draw a sketch of one of the shapes the class created with the drips from the swinging pendulum in Lesson 8.1.1.

a. Sketch what the shape would look like if the bag were pulled out farther before it was let go.

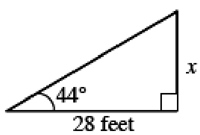
b. Sketch what the shape would look like if the paper underneath the pendulum traveled faster.

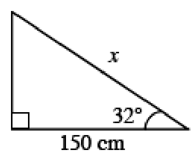
8-4. Karin was working on graphing the function $f(x)=\frac{2}{x-3}$. She made a table (shown below), but she is not sure how to graph the values in the table. Show Karin how to make her graph and tell her everything you know about her function.

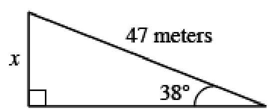
x	-3	-2	-1	0	1	2	3	4	5	6	7	8	9
$f(x)$	$-\frac{1}{3}$	$-\frac{2}{5}$	$-\frac{1}{2}$	$-\frac{2}{3}$	-1	-2	*	2	1	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{1}{3}$

*undefined

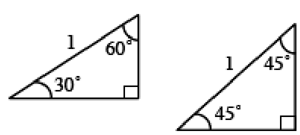
8-5. In each of the following triangles, find the length of the side labeled x.

a.


b.


c.


8-6. Copy the triangles at right and label the missing side lengths.



8-7. Find the equation of the parabola that passes through the points (0, 0), (3, 9), and (6, 0).

8-8. Solve and check your solution: $2\sqrt{21-x}-\sqrt{3x-6}=5$.

8-9. Find the inverse functions for the functions given below.

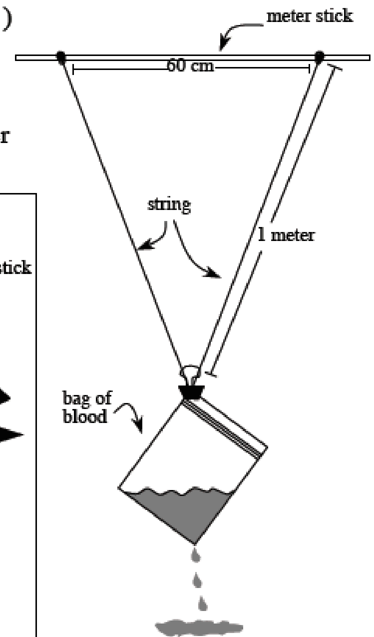
a. $f(x)=\sqrt[3]{4x-1}$

b. $g(x)=\log_7 x$

8-10. Find the x- and y-intercepts of $y-7=3^{(x+4)}$.

8-11. Change $x^2-2x+y^2-29=0$ to graphing form then sketch the graph and label the important points.

Directions for the Pendulum Experiment (Problem 8-1)



Find a space in the classroom where there is room to work. You may need to move your table or desks out of the way.

Set up your pendulum by attaching your bag of liquid to a meter stick using string, as shown at right.

Assign the following tasks:

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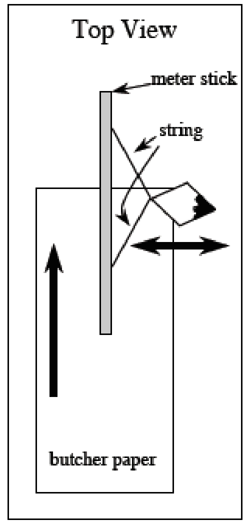

Tips for a successful experiment:

- The student holding the meter stick should hold the places where the string attaches. Otherwise, the string has a tendency to slide toward the center of the meter stick.
- Use as much of the string as you can so that your pendulum is as long as possible.
- When you are ready, have your teacher cut a *very* small hole in the corner of the bag. The student holding the bag should pinch this hole closed until the pendulum begins swinging.
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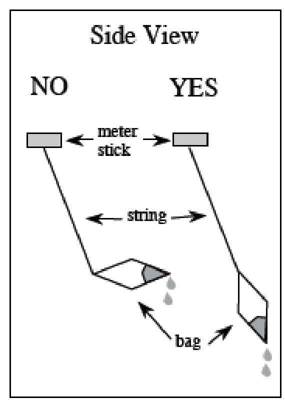


8.1.2 How can I graph it?

Graphing the Sine Function

Today you will use what you know about right-triangle relationships and about graphing functions to **investigate** a new function.

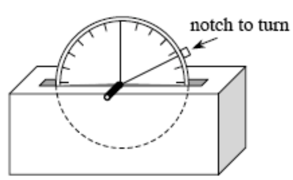
8-12. "*HURRY!!! Let's get there before the line gets too long*!" shouts Antonio to his best friend René as they race to get on *The Screamer*, the newest attraction at the local amusement park.

"*It's only been open for one day, and already everyone is saying it's the scariest ride at the park!*" exclaims Antonio. "*I hear they really had to rush to get it done in time for summer*."

Antonio whistles as he screeches to a halt in front of the huge sign that says, "*Welcome to The Screamer, the Scariest Ride on Earth.*" The picture below it shows an enormous wheel that represents *The Screamer*, with its radius of 100 feet. Half of the wheel is below ground level, in a very dark, murky pit with water at the bottom. As *The Screamer* rotates at dizzying speeds, riders fly up into the air before plunging downward through blasts of freezing air, hair-raising screams, and sticky spider webs into the pit where they splash through the dark, eerie water on their way back above ground.

René and Antonio wait impatiently to get on the ride, watching passengers load and unload. New passengers get on and strap themselves in as others emerge from the pit looking queasy. The ride rotates 15° to load and unload the next set of riders. As René straps himself in, he remembers Antonio's ominous words: "*I hear they really had to **rush** to get it done in time for summer*."

Sure enough, just as the ride plunges René and Antonio into the greasy water, they hear the piercing scream of metal twisting. Sparks fly and the pit fills with smoke as the ride grinds to a halt. To escape, all of the passengers must climb vertically to ground level from wherever they got stuck, either up from the pit or down from dizzying heights.



Your task: Find a function that describes the distance each passenger must climb in order to escape from the broken ride, *The Screamer*.

To help you gather data, your teacher will provide you with the materials to build a model of this situation. Layer your transparency circle on top of your cardboard circle and insert a toothpick to act as an axle. Then slide the circles into the slit in your cardboard box, as shown in the diagram at right.

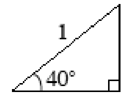
a. Use the transparent One Unit Ruler to measure the escape heights on the model for at least 16 different possible seat positions. The seat could be in the pit, high in the air, or right at ground level. Remember that this position depends on the angle of the ride's rotation when it broke down. The radius of the model wheel will be referred to as one unit. That is, a model height of one unit corresponds to an actual height of 100 feet on the ride, so an actual height of 80 feet on the ride will be represented by a measured height of 0.8 units on the model. Record your data in a table like the one shown below. Leave room for additional columns.

Degree of Rotation from 0° (Platform)	Measured Height $(0 \le y \le 1)$	Actual Height Above (or Below) Platform	

b. Graph your data on a large graph.

c. Suppose you were asked to add 20 more data points to your table. What shortcuts could you use to reduce the amount of work?

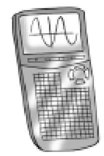
8-13. What function can you use to model the situation in problem 8-12? To help you figure it out, sketch the right triangle shown in the diagram at right.



a. With your team, write an equation and use it to calculate the height of the triangle. Does the escape height you calculated seem reasonable compared to the data you collected in problem 8-12?

b. Write an equation representing the escape height $h(\theta)$ for *any* passenger, for any angle of rotation of *The Screamer*. (Note that the angle is represented by the Greek letter theta, which looks like this: θ.)

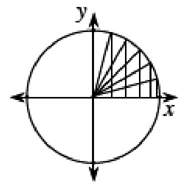
c. Enter the data from the first two columns of your table into your grapher. Adjust the viewing window so you can see all of the data. Then graph $h(\theta)$ on top of the data. How well does $h(\theta)$ fit your data?



d. Adjust the viewing window so that you can see more of the graph of $h(\theta)$. Describe the behavior of the graph as θ gets larger. Does this make sense? Why or why not?

e. Use the 'table' function of your calculator to find its calculated values for $h(\theta)$. Add another column to your table from problem 8-12, label it with the equation you found for $h(\theta)$, and enter these values, rounding off to the nearest hundredth. How do the calculated values compare with your measured ones?

8-14. René and Antonio finally make it home from the amusement park unhurt, but in need of a shower. As soon as they have cleaned up, they go over to a friend's house to share their scary experience on *The Screamer*. They draw a picture of the Ferris wheel and five of the seats, located at $15°$, $30°$, $45°$, $60°$, and $75°$ (shown at right and on the resource page provided by your teacher).

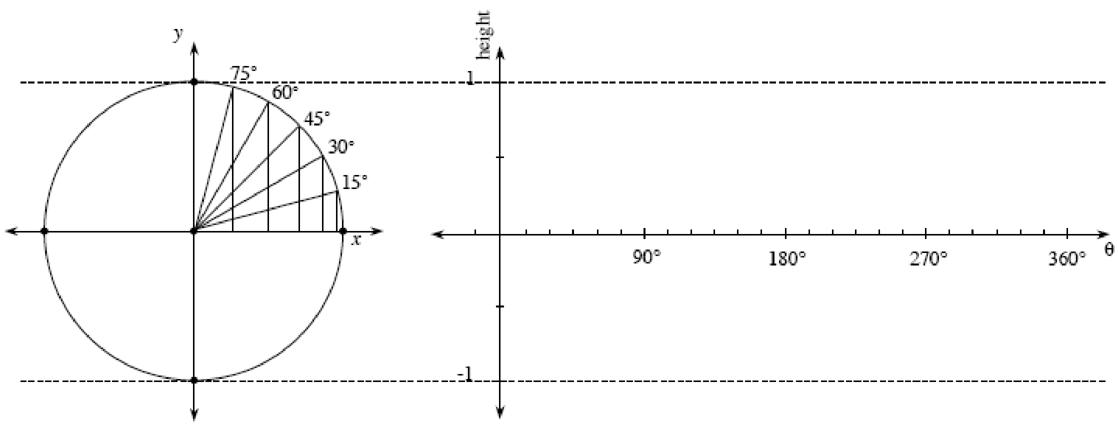


a. Label each triangle with its *calculated* height. You can use your data from problem 8-12. If you do not have data for all of these angles, return to the 'table' function on your grapher. Plot these heights at their angle location on the coordinate system to the right of the circle. You will be plotting points in the form (x = angle in degrees, y = height).

b. Draw five new triangles that are congruent to the first five, but that are located in the second quadrant. Label these with their angle measures (from 0°) and heights. Use the angle measures and heights to plot five new points on the graph that correspond to these five new points on the circle.

c. Continue this process by drawing triangles in the third and fourth quadrants. You should have a total of twenty triangles drawn and twenty points plotted. Then label the points where the circle intersects the x- and y-axes with their angle measures and heights and then add points for them to the graph as well. Sketch a smooth curve through the points.

d. Discuss with your team all of the relationships you can find among the points on the circle and between your unit circle and the graph. Be prepared to share your ideas with the class.



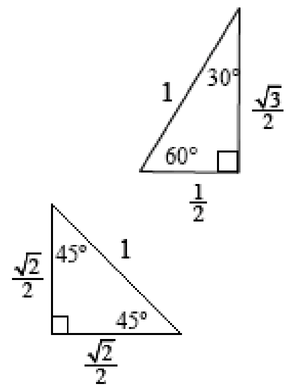
MATH NOTES

METHODS AND MEANINGS

Special Right-Triangle Relationships

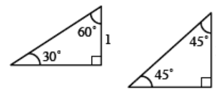
As you may recall from geometry, there are certain right triangles whose sides have special relationships that make certain calculations easier. One such triangle is half of an equilateral triangle and is known as a **$30° – 60° – 90°$ triangle**, named after the degree measures of its angles.

The other special triangle is half of a square and is known as the **$45° – 45° – 90°$ triangle**. Both triangles are shown at right.



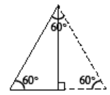
Review & Preview

8-15. Copy the triangles at right.

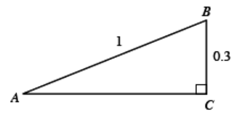


a. Label the missing sides with their *exact* lengths. That is, leave your answers in radical form.

b. The $30° - 60° - 90°$ triangle is sometimes called a half-equilateral. Draw a picture to illustrate this, and explain how that fact can be used to help label the missing sides in part (a).



8-16. Find the measure of angle *A* in the diagram at right.



8-17. This is a Checkpoint problem for completing the square and finding the vertex of a parabola.

Complete the square to change the equation $y = 2x^2 - 4x + 5$ to graphing form, identify the vertex of the parabola, and sketch its graph.

If you needed help completing the square to change this equation to graphing form and identify the vertex, then you need more practice. Review the Checkpoint 14 materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to complete the square and identify the point (h, k) for equations such as these quickly and accurately.

8-18. Leadfoot Lilly was driving 80 miles per hour when she passed a parked highway patrol car. By the time she was half a mile past the spot where the patrol car was parked, the officer was driving after her at 100 miles per hour. If these rates remain constant, how long will it take the officer to catch up to Lilly? Write and solve an equation to represent this situation.

8-19. Evaluate each expression without using a calculator or changing the form of the expression.

a. $\log(1)$

b. $\log(10^3)$

c. $10^{\log(4)}$

d. $10^{3\log(4)}$

8-20. Complete the table of values for $f(x) = \frac{x^2+4x-5}{x-1}$.

x	-2	-1	0	1	2	3
y						

a. Graph the points in the table. What kind of function does it appear to be? Why is it not correct to connect all of the dots?

b. Look for a simple pattern for the values in the table. What appears to be the relationship between *x* and *y*? Calculate $f(0.9)$ and $f(1.1)$ and add the points to your graph. Is there an asymptote at $x = 1$? If you are unsure, calculate $f(0.99)$ and $f(1.01)$ as well.

c. Simplify the formula for $f(x)$. What do you think the complete graph looks like?

8-21. In 1998, Terre Haute, Indiana had a population of 72,000 people. In 2000, the population had dropped to 70,379. City officials expect the population to level off eventually at 60,000.

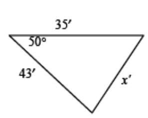
a. What kind of function would best model the population over time?

b. Write an equation that would model the changing population over time.

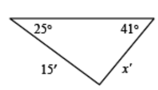
8-22. A semi-circular tunnel is 26 feet high at its highest point. A road 48 feet wide is centered under the tunnel. Bruce needs to move a house on a trailer through the tunnel. The load is 22 feet wide and 24 feet high. Will he make it? Use a diagram to help **justify** your reasoning completely.

8-23. Find the value of *x*.

a.



b.



8-24. Solve the system of equations shown at right.

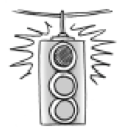
$$x + y + z = 40$$
$$y = x - 5$$
$$x = 2z$$

8-25. What is the domain of the entire graph of $h(\theta)=\sin\theta$? **Justify** your reasoning.

8-26. Antonio's friend Jessica was also on *The Screamer* when it broke. Her seat was 65 feet above the ground. What was her seat's angle of rotation? Is there more than one possibility?

8-27. Hilda was working on her homework. She completed the square to change $y=3x^2-24x+55$ to graphing form in order to identify the vertex of the parabola. She did the work at right and identified the vertex to be $(4, 39)$.

$$y=3x^2-24x+55$$
$$y=3(x^2-8x)+55$$
$$y=3(x^2-8x+16)+55-16$$
$$y=3(x-4)^2+39$$



When she got back to class and checked her answers, she discovered that the vertex she found is incorrect, but she cannot find her mistake. Examine Hilda's work and explain to her what she did wrong. Then show her how to complete the square correctly and identify the vertex.

8-28. Consider the system of equations at right.

$$x^2+y^2=25$$
$$y=x^2+3$$

a. Solve the system graphically.

b. Now solve the system algebraically. In this case, which variable is easier to solve for first? Do it. Now try to solve for the other variable first. What caused you to get stuck?

8-29. Mr. Keis wrote the following problem on the board and told his class, *"No calculators please. Simplify. You have sixty seconds!"*

$$\left(\frac{13^{12}}{14^{23}}\right)\left(\frac{27^3}{13^{11}}\right)\left(\frac{2^{10}}{27^4}\right)\left(\frac{14^{22}}{13}\right)\left(\frac{27}{2^9}\right)$$

Time yourself and simplify the expression. Did you meet the challenge?

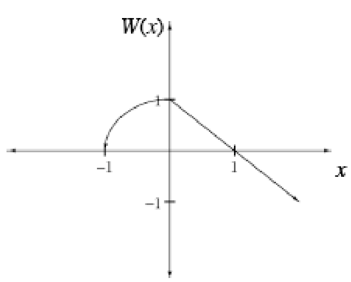
8-30. Graph the system at right.

$$1+x-y\ge 3x-2y-4$$
$$y<2x^2+1$$

8-31. Solve each of the following equations and check your solutions.

a. $\frac{1}{x}+\frac{5}{4x}=\frac{1}{x+1}$ Challenge: $\frac{1}{1-x}+\frac{1}{1+\sqrt{x}}=\frac{1}{1-\sqrt{x}}$

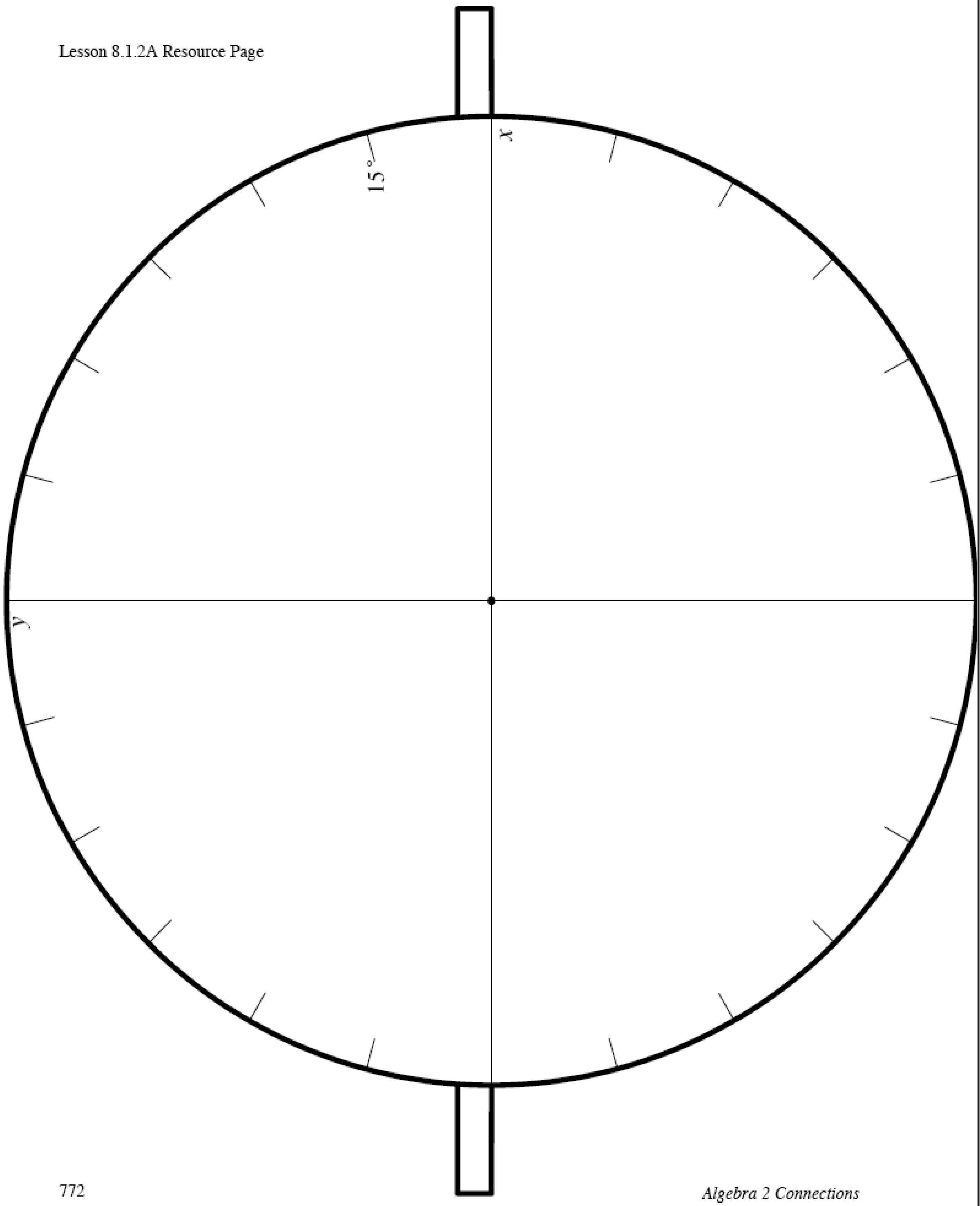
8-32. A function $W(x)$ is sketched at right.

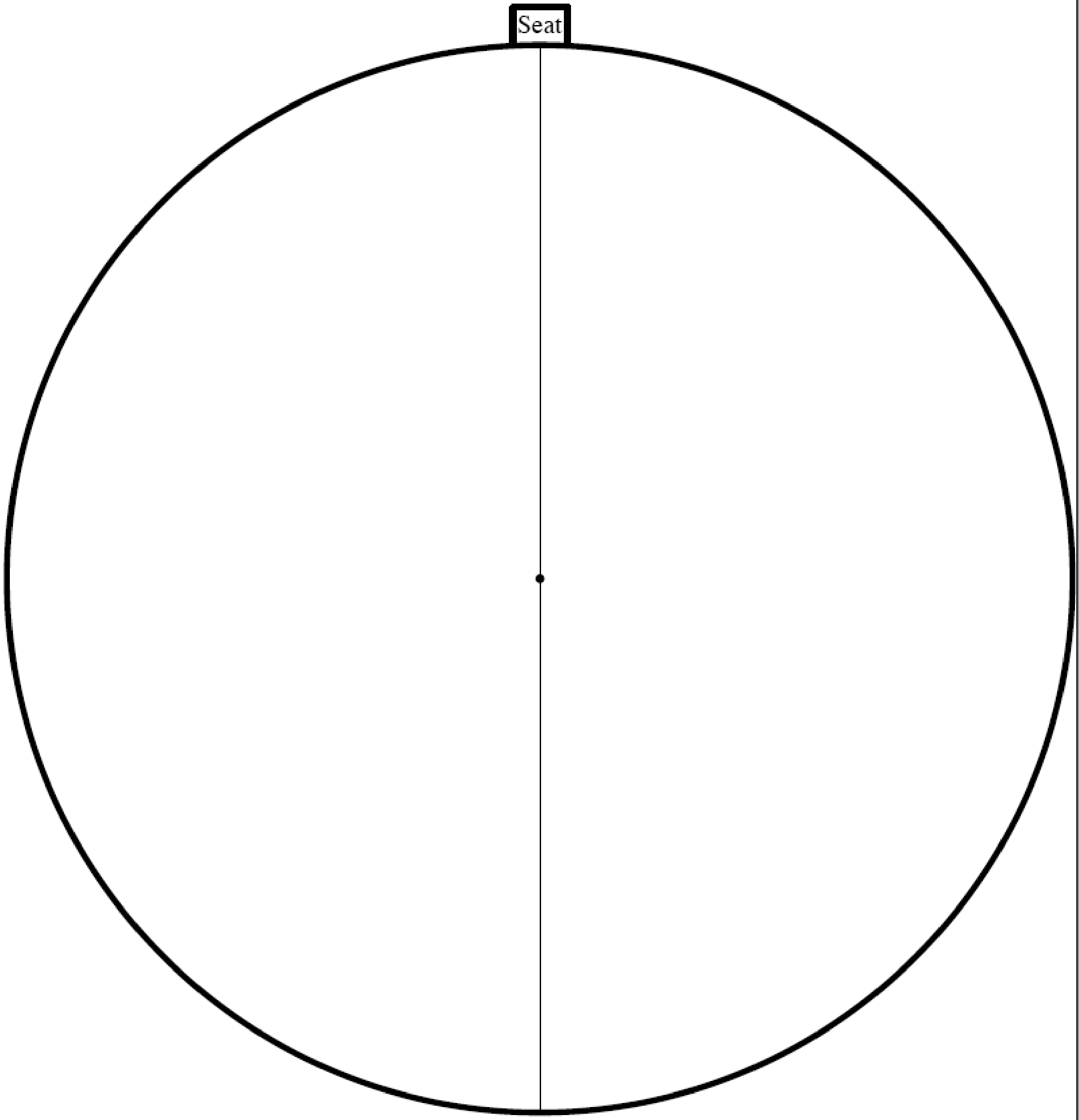


a. Make your own copy of the graph, and then sketch the graph of the inverse of $W(x)$.

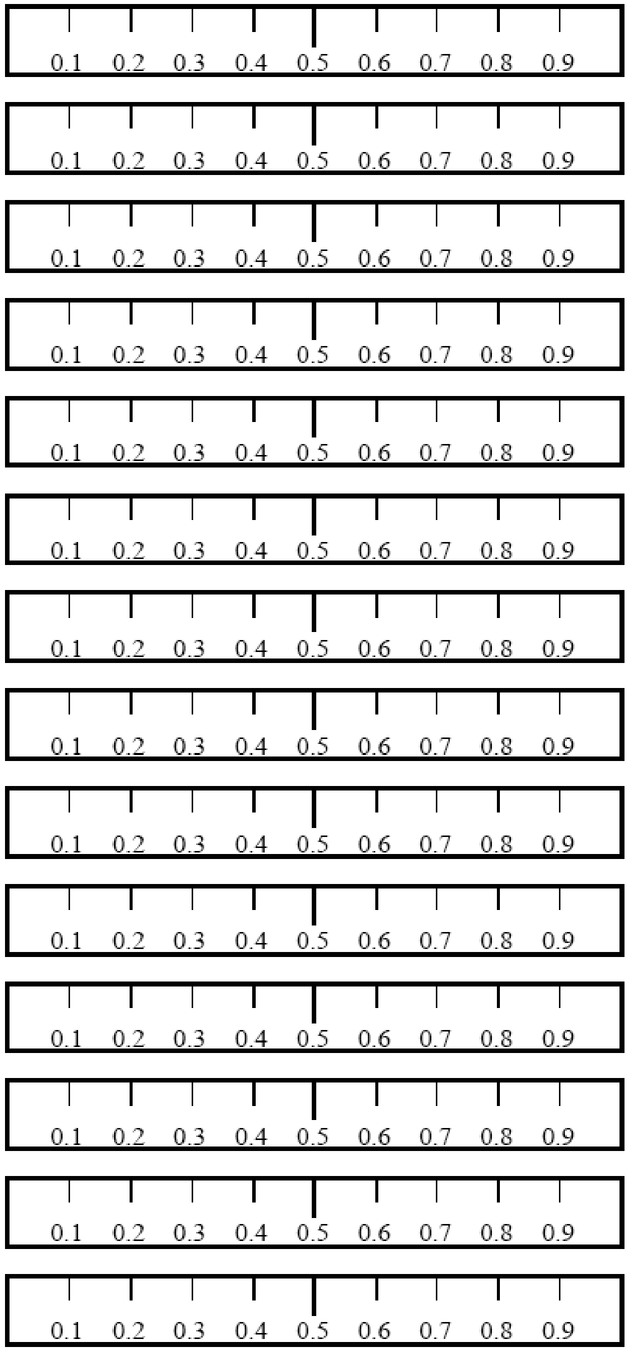
b. Is the inverse a function? Explain.

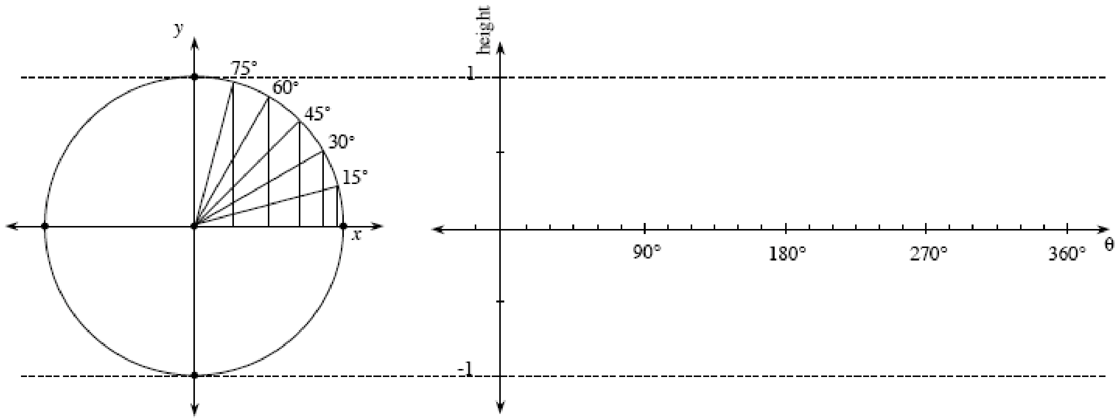
8-33. Mary has an antique marble collection containing 40 marbles. She has five more red marbles than blue and twice as many red as green marbles. Write a system of equations and use matrices to find the number of each color of marble.


15°
x
y

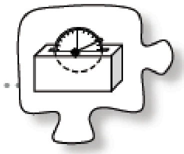

Seat

Lesson 8.1.2C Resource Page



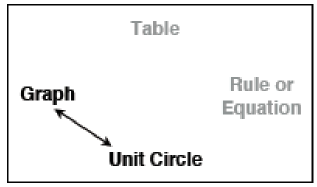

y
x
75°
60°
45°
30°
15°
height
1
-1
90°
180°
270°
360°
θ

8.1.3 How are circles and sine graphs connected?



Unit Circle ↔ Graph

Throughout this course, you have used multiple representations (table, graph, equation, and situation) to solve problems, **investigate** functions, and **justify** conclusions. In Lesson 8.1.2 you found that a unit circle is one representation of a sine function. Today you will **investigate** the connections between the unit circle and the graph, as you build a deeper understanding of the sine function.



8-34. Draw a circle on your paper. Draw a triangle that could represent René and Antonio's position on the wheel when *The Screamer* came to a sudden stop. Be sure to choose a different angle position from any of those you drew in problem 8-14.

a. Label the triangle with its height and its angle measure (from $0°$).

b. Did any other riders have to climb the same distance to get to safety (up *or* down) as René and Antonio did? If so, draw the corresponding triangles and label them completely.

c. What is the relationship between these triangles? Work with your team to **generalize** a method for finding all of the other corresponding angles when you are given just one angle.

8-35. In problem 8-34, you used a unit circle to find the height of a seat on *The Screamer*. Could you use your graph of $y = \sin\theta$ instead to find the height?

a. Use the Lesson 8.1.3 Resource Page (a sine calculator) provided by your teacher to find the height of a seat that had rotated 130° from the starting platform.

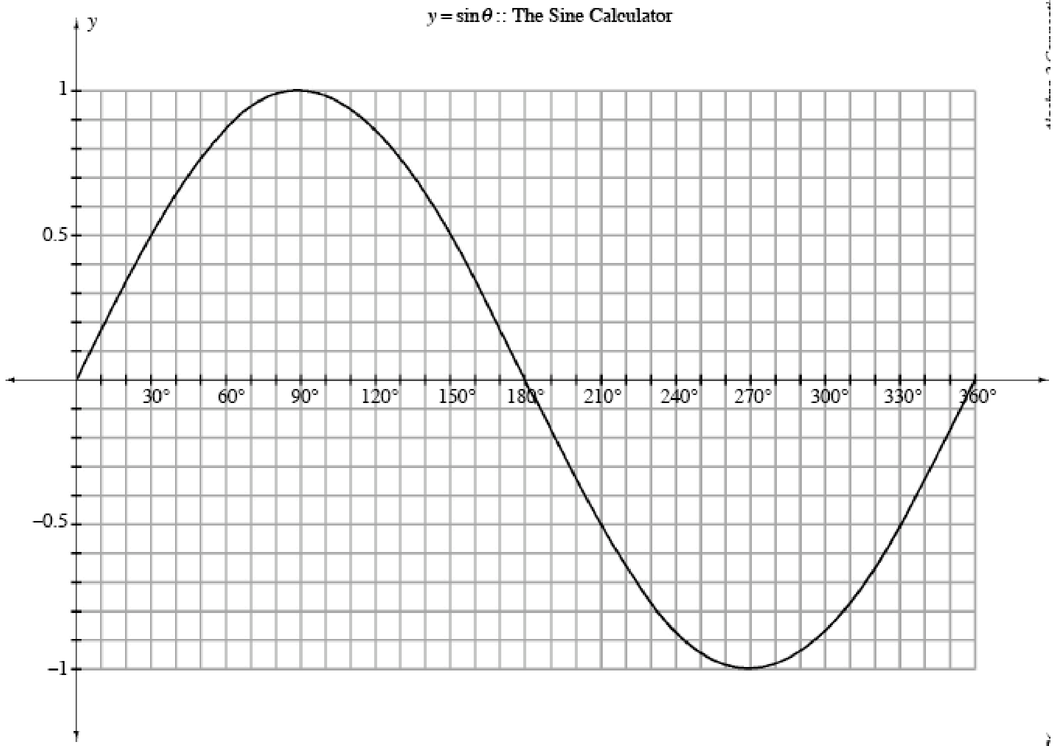
b. Are there any other seats at exactly the same height? If so, indicate them on your sine calculator.

c. How can you use the symmetry of the graph to calculate which angles correspond to seats with the same height? Discuss this with your team and be prepared to share your **strategies** with the class.

d. For each of the following angles, use the sine calculator from the resource page to find the height at that angle and to find another angle with the same height. Then sketch a small unit circle, draw in each pair of angles, and label the heights.

i. 80° *ii.* 200° *iii.* 310°

$y = \sin\theta$ :: The Sine Calculator



Lesson 8.1.3 Resource Page

8-36. With your team, discuss the ways in which a unit circle and the graph of $s(\theta) = \sin\theta$ are connected. Be prepared to share your ideas with the class. Then record your ideas in a Learning Log. Use diagrams, arrows, and other math tools to help demonstrate your ideas. Label this entry "Unit Circle $\leftrightarrow$ Graph for $s(\theta) = \sin\theta$" and label it with today's date.

Review & Preview

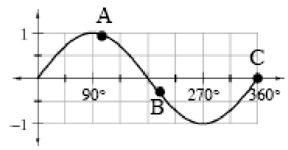
8-37. Sketch a graph of the first two cycles of $s(\theta) = \sin\theta$. Then label your graph to show the following positions of a passenger on *The Screamer*.

a. The passenger gets on initially.

b. The passenger reaches the bottom of the water pit.

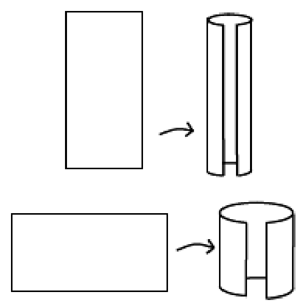
c. The passenger is halfway between the highest point of *The Screamer* and the ground level.

8-38. Each of the points on the graph at right represents the position of a rider on *The Screamer*. Draw a diagram of each rider's position on a unit circle and describe where the rider is.



8-39. While trying to measure the height of a tree, Julie noticed that a 3.5-foot post had a 4.25-foot shadow. If the tree's shadow is 100 feet long, how tall is the tree?

8-40. Suppose you were to bend two whole sheets of $8\frac{1}{2}$-by-11 paper to form two cylinders (a tall, skinny one and a short, wide one). The volume of a cylinder can be calculated using the formula $V = (\textit{base area})(\textit{height})$ or $V = \pi r^2 h$.



a. Do the two cylinders have the same volume? **Justify** your answer.

b. Is the result different if you start with $8\frac{1}{2}$-by-$8\frac{1}{2}$ paper?

8-41. Find the x- and y-intercepts of the quadratic function $y = 3x^2 + 6x + 1$.

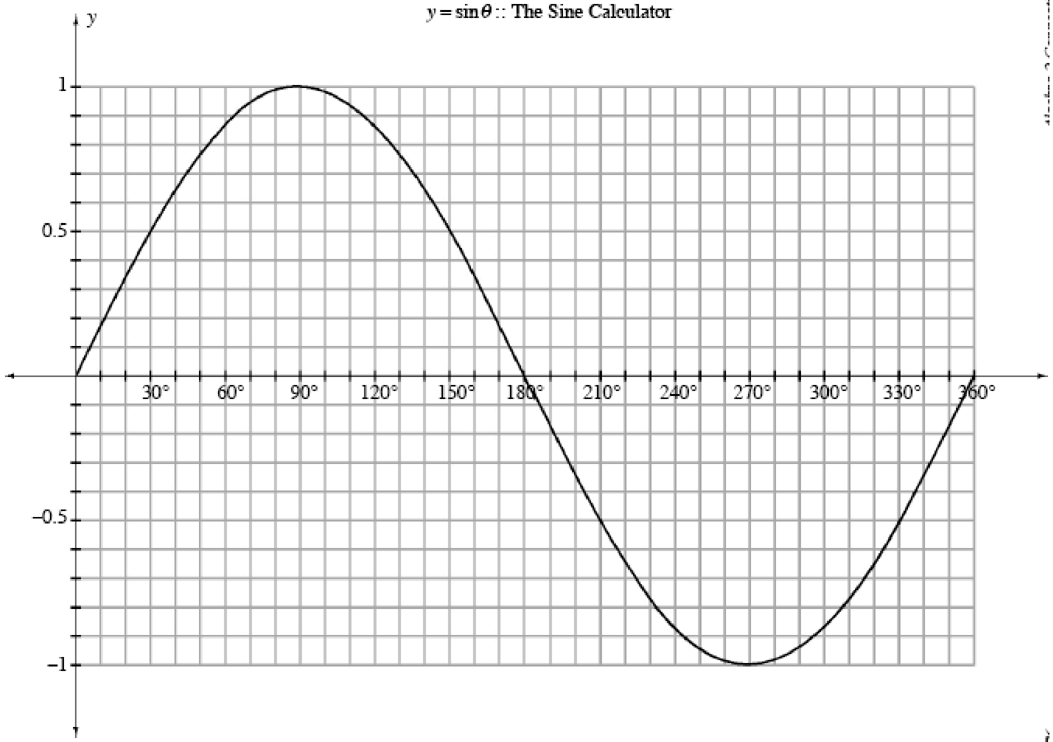
8-42. Write the quadratic function in problem 8-41 in graphing form and sketch its graph.

8-43. Solve $|x+5| = |x| - 5$ by graphing. Express your solution algebraically.

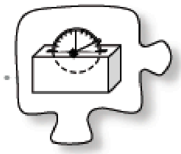
8-44. Solve $\log_2 x = 2^x$ using any method.

8-45. Maria Elena is collecting college pennants. She has five fewer pennants from Washington campuses than from California campuses and twice as many pennants from California campuses as from Pennsylvania campuses. She has 40 pennants in her collection. Write a system of equations to find the number of Pennants from each state.

$y = \sin\theta$ :: The Sine Calculator



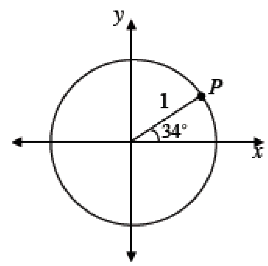
8.1.4 How can I graph cosine?



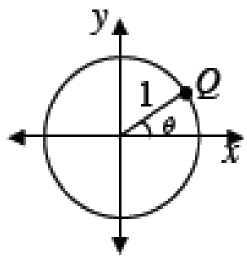
Graphing and Interpreting the Cosine Function

In this lesson, you will use your knowledge of right triangles again—this time to develop your understanding of another cyclic function.

8-46. Work with your team to find the coordinates of point *P* on the unit circle shown at right. Is there more than one way to find point *P*? Be prepared to share your **strategies** with the class.



8-47.



Now **generalize** what you found in problem 8-46 to write the coordinates of point Q on the unit circle shown at left.

8-48. What can a sine value tell you about a point on a circle? What about a cosine value?

8-49. If you know the sine of an angle in a unit circle, can you find its cosine? How? Work with your team to find a **strategy** and be prepared to share it with the class.

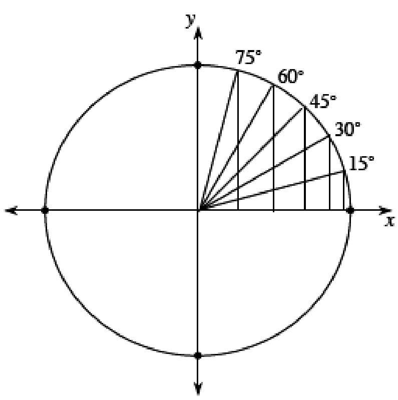
8-50. Obtain a copy of the Lesson 8.1.4A Resource Page from your teacher. To graph the cosine function you will follow the same process as you did for the graph of the sine function in problem 8-14, but use the horizontal distance (or base of the triangle) instead of the height.

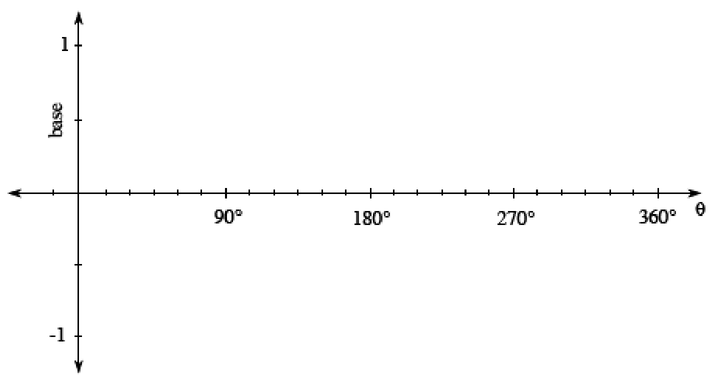
a. Label the length of the base of each triangle in the unit circle. Plot these lengths at their angle location on the coordinate system to the right of the circle. You will be plotting points in the form (angle in degrees, base).

b. Draw five new triangles that are congruent to the first five, but that are located in the second quadrant. Label the angle measure (from 0°) and the base for each triangle. Add five new corresponding points to the graph.

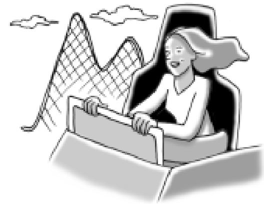
c. Continue this process by drawing triangles in the third and fourth quadrants. You should have a total of twenty triangles drawn and twenty points plotted. Then find the four points where the circle crosses the axes and label them with both their angle measures and their horizontal distances from the origin. Add points for these to the graph on the right as well. Finally, sketch a smooth curve through the points.

d. Compare this graph to the sine graph you got from graphing heights in problem 8-14. How are the two graphs similar? How are they different?





8-51. Remember the scary Ferris wheel, *The Screamer*? LaRasha does! She was riding *The Screamer*, sitting 27 horizontal feet away from the central support pole, when the ride stopped. What was her seat's angle of rotation? Is there more than one possibility? **Justify** your answer using as many representations as you can.



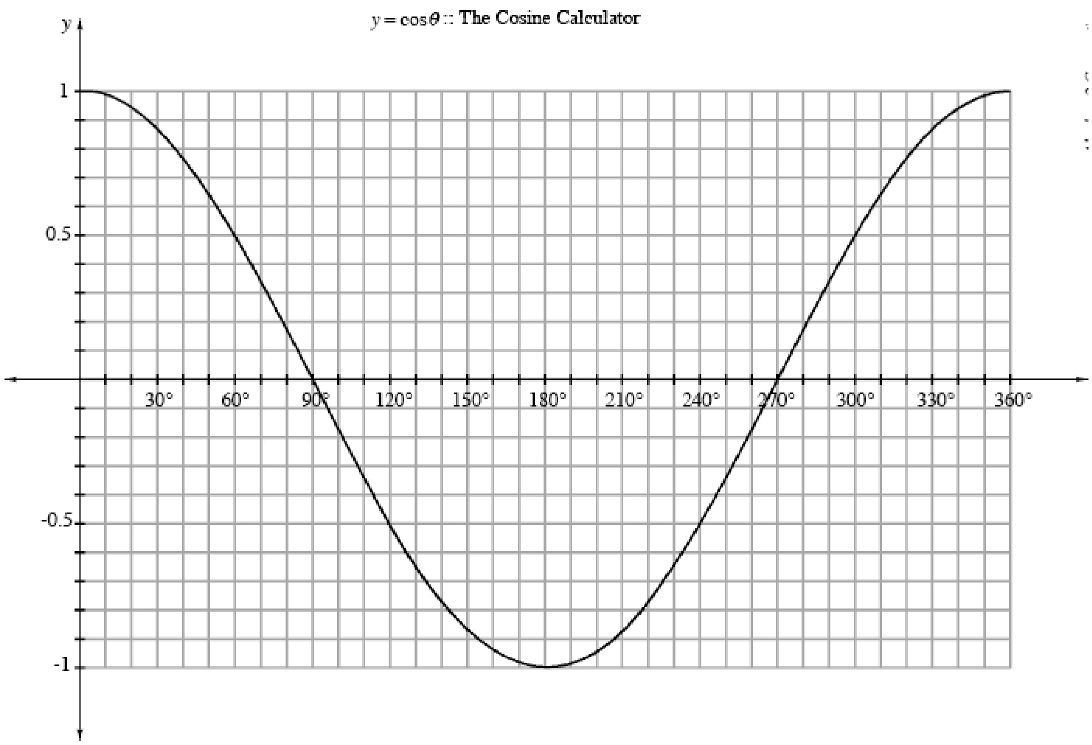
8-52. UNIT CIRCLE ↔ GRAPH

In problem 8-51, did you use a graph of $y = \cos\theta$ to find lengths of bases of triangles?

a. Use the Lesson 8.1.4B Resource Page (a cosine-calculator graph) provided by your teacher to find the length of the base of a triangle formed by a seat on *The Screamer* that had rotated 130° from the starting platform.

b. Are there any other triangles with the same base? If so, mark their corresponding points on your cosine calculator.

c. How can you use the symmetry of the cosine-calculator graph to calculate the angle location of seats on *The Screamer* that have the same base? Is your method different than the one you used to find the heights?

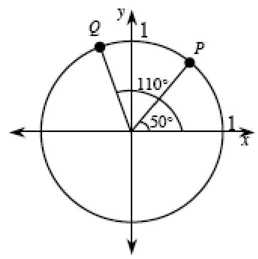


Lesson 8.1.4B Resource Page

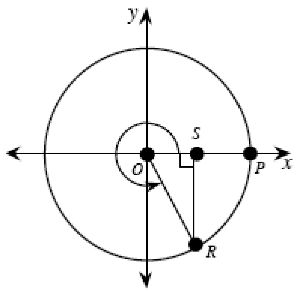
April 02, 2011

Review & Preview

8-53. Find the coordinates of points P and Q on the unit circle at right.



8-54. The measure of $\angle ROS$ in ΔROS below is 60°.



a. The curved arrow represents the rotation of $\overline{OR}$, beginning from the positive x-axis. Through how many degrees has $\overline{OR}$ rotated?

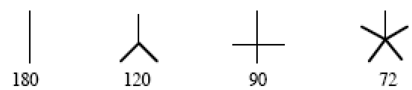
b. If $OR = 1$, what are the *exact* length of OS and SR?

c. What are the *exact* coordinates of point R?

8-55. What angle in the first quadrant could you reference to help you find the sine and cosine of each of the following angles?

a. 330° b. 120° c. 113° d. 203°

8-56. At right is a sequence of doodles and some numbers to go with them.



a. Draw the next two doodles and state the numbers to go with them.

b. What number would go with the 99th doodle? With the n^{th} doodle?

8-57. Solve $\frac{1}{1-\sqrt{x}} = 1 + \frac{\sqrt{x}}{1-\sqrt{x}}$.

8-58. Solve $(\frac{1}{8})^{(2x-3)} = (\frac{1}{2})^{(x+2)}$.

8-59. Sketch a graph of each equation below.

a. $y = -2(x-2)^2 + 3$ b. $y = (x-1)^3 + 3$

8-60. Solve $3^x + 5 = x^2 - 5$ using any method.

8-61. Rip-Off Rentals charges \$25 per day plus 50¢ per mile to rent a mid-sized car. Your teacher will rent you his or her family sedan and charge you only 3¢ if you drive one mile, 6¢ if you drive two miles, 12¢ if you drive three, 24¢ for four, and so on.

a. Write a rule that will give you the cost to rent each car.

b. If you plan to rent the car for a two-day road trip, which is the better deal if you drive 10 miles? 20 miles? 100 miles?

8-62. Refer back to your solutions from problems 8-24, 8-33, and 8-45. Explain how these problems are related.

8-63. Shinna was riding *The Screamer* when it broke down. Her seat was 53 horizontal feet from the central support pole. What was her seat's angle of rotation? How can you tell?

8-64. Sketch a unit circle. In your circle, sketch in an angle that has:

a. A positive cosine and a negative sine.

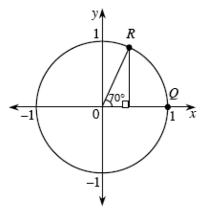
b. A sine of -1.

c. A negative cosine and a negative sine.

d. A cosine of about -0.9 and a sine of about 0.4.

e. Could an angle have a sine equal to 0.9 and cosine equal to 0.8? Give an angle or explain why not.

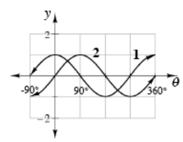
8-65. A 70° angle is drawn for you in the unit circle at right.



a. Approximate the coordinates of point R.

b. How could you represent the *exact* coordinates of point R?

8-66. Daniel sketched the graphs at right for $y = \sin\theta$ and $y = \cos\theta$.



Unfortunately, he forgot to label the graphs, and now he cannot remember which graph goes with which equation. Explain to Daniel how he can tell (and remember!) which graph is $y = \sin\theta$ and which is $y = \cos\theta$.

8-67. Consider the system of equations $y = \cos x$ and $y = -1$.

a. Is it possible to solve this system by substitution? By the Elimination Method? By graphing?

b. List at least five possible solutions.

c. Consider the list of solutions you wrote in part (b) as a sequence and write a rule to represent *all* possible solutions.

8-68. This problem is a checkpoint for writing and solving exponential equations.

When rabbits were first brought to Australia, they had no natural enemies. There were about 80,000 rabbits in 1866. Two years later, in 1868, the population had grown to over 2,400,000!

a. Why would an exponential equation be a better model for this situation than a linear one? Would a sine function be better or worse? Why?

b. Write an exponential equation for the number of rabbits t years after 1866.

c. How many rabbits do you predict would have been present in 1871?

d. According to your model, in what year was the first pair of rabbits introduced into Australia? Is this reasonable?

e. Actually, 24 rabbits were introduced in 1859, so the model is not perfect, but is close. Is your exponential model useful for predicting how many rabbits there are now? Explain.

f. Check your answers to parts (a) through (e) by referring to the Checkpoint 15 materials at the back of your book.

If you needed help to complete any part of this problem, then you need more practice writing and solving exponential equations to solve problems. Review the Checkpoint 15 materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to write and solve exponential equations to solve problems quickly and accurately.

8-69. Solve each equation.

a. $\frac{3}{x+1} = \frac{4}{x}$

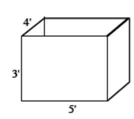
b. $\frac{3}{x+1} + \frac{4}{x} = 2$

c. $\frac{3}{x+2} + 5 = \frac{3}{x+2}$

d. Explain why part (c) has no solution.

8-70. Write the equation of any line parallel to $3x - y = 2$. Then write the equation of a parabola that intersects your line but does *not* intersect the graph of $3x - y = 0$.

8-71. A $5' \times 4' \times 3'$ box is made for the purpose of storage. What is the longest pole that can fit inside the box?



8-72. While working on their homework on sequences, Davis was suddenly stumped!

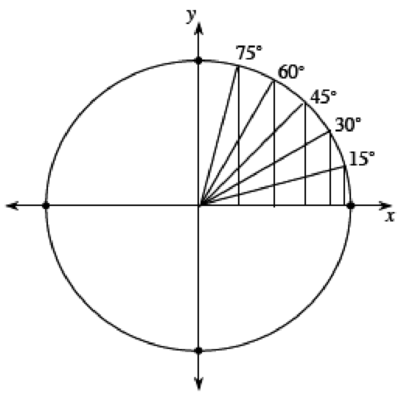
"This problem doesn't make sense!" he exclaimed. Tess was working on her homework as well.

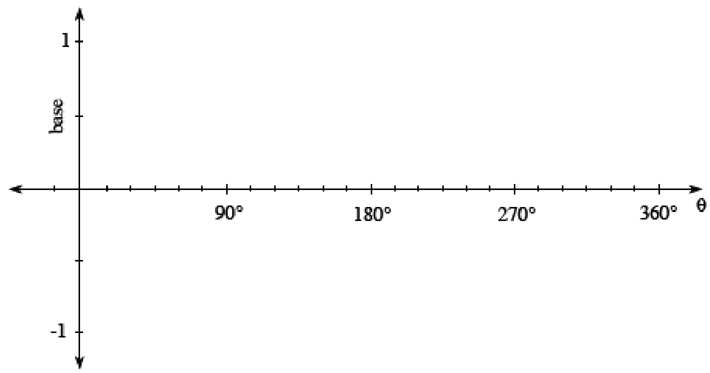
"What's the problem?" she asked.

"This problem is about a SEQUENCE, $t(n) = 9n - 2$*, but it is asking whether or not it is a function. How can a sequence be a function?"*

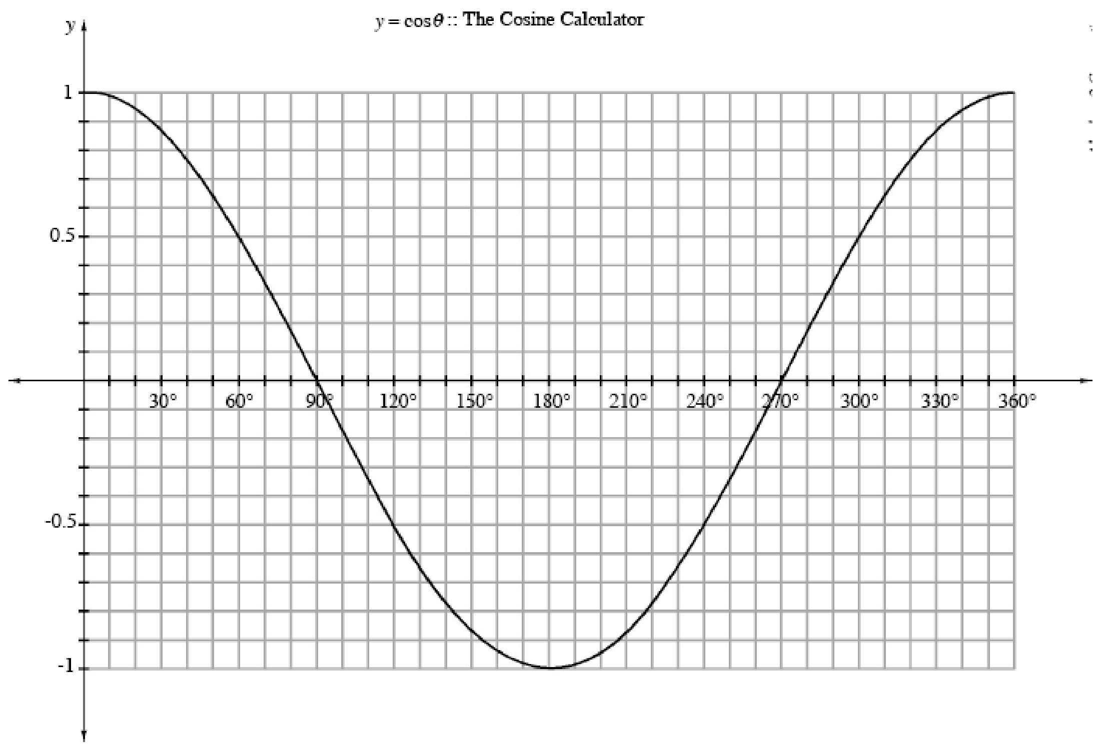
"Well of course a sequence is a function!" said Tess.

Who is right? Should Davis be confused, or is Tess correct? Explain completely.



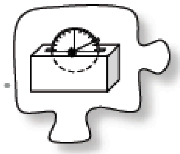


Lesson 8.1.4A Resource Page



Lesson 8.1.4B Resource Page

8.1.5 How else can I measure angles?



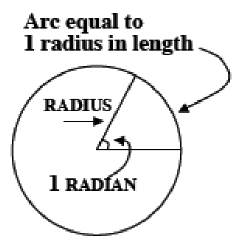
Defining a Radian

Whose idea was it to measure angles in degrees? And why are there 360° in a full turn? This decision actually dates back almost 4000 years! Degrees were created by the Babylonians, an ancient people who lived in the region that is now Iraq. The Babylonians also based their number system, called a sexigesimal system, on sixty.

Although you are familiar with measuring angles in degrees, this is not the only way to measure angles, nor is it necessarily the most useful. Today you will learn a different unit for measuring angles called a **radian**. Using radians instead of degrees is actually the standard across mathematics! When you take calculus, you will learn why radians are used in math more often than degrees.

8-73. What word are you reminded of when you hear the word **radian**? Discuss this with your team and make a conjecture about how this might relate to a way to measure angles. Be prepared to share your ideas with the class.

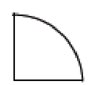
8-74. HOW TO MAKE A RADIAN



Imagine wrapping the radius of a circle around the circle. The angle formed at the center of the circle that corresponds to the arc that is one radius long has a measure of exactly one **radian**.

Your teacher will provide each member of your team with a differently sized circular object and some scissors.

a. Trace your circular object onto a sheet of paper and carefully cut out the circle. Fold the paper circle in half and then in half again so that it is in the shape of a quarter circle, as in the diagram at right. How can you see the radius of your circular object in this new folded shape?



b. Place your circular object onto another sheet of paper and trace it again, only this time leave the circular object in place. Roll (or wrap) a straight edge of your folded circle around your circular object and mark one radius length on the traced circle. Then mark another radius length that begins where the first one ended. Continue marking radius lengths until you have gone around the entire circle.

c. Remove the circular object from your paper. On your traced circle, connect each radius mark to the center, creating central angles. Each angle you see, formed by an arc with a length of one radius, measures one **radian**. Label each of the radius lengths and each angle that measures one full **radian**. Write a short description of how you constructed an angle with measure one radian.

8-75. Assume the radius of a circle is one unit.

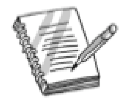
a. What is the area of the circle? What is its circumference?

b. How many radii would it take to wrap completely around the circle? Express your answer as a decimal approximation *and* as an exact value.

c. Does the size of the circle matter? That is, does the number of radii it takes to wrap around the circle change as the radius of the circle gets larger or smaller? Why does this make sense?

d. Exactly how many radians are in 360°? In 90°?

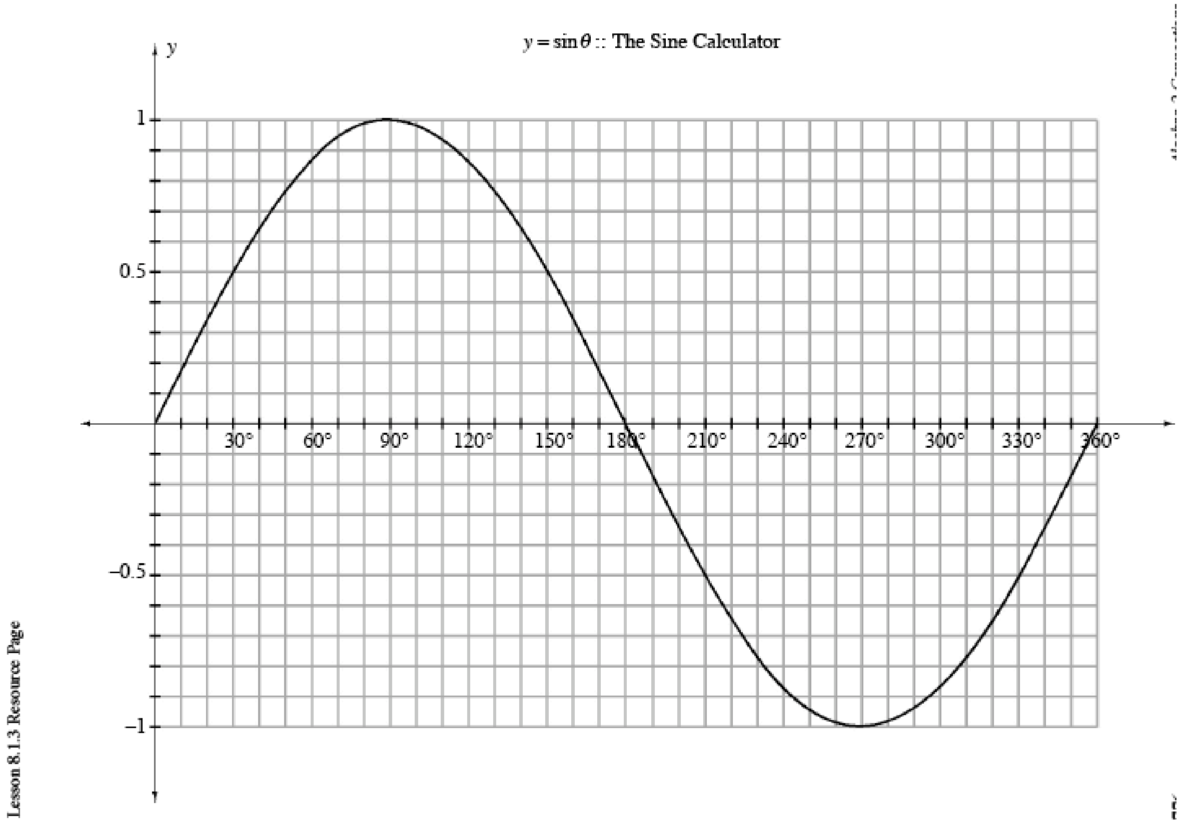
e. How is a radian related to a radius? Explain your understanding of this relationship in your Learning Log. Use diagrams to support your explanation. Title this entry "Radians" and label it with today's date.



8-76. Parts (a) through (g) below describe angles. Draw each angle on its own unit circle.

a. 1 degree
b. 1 radian
c. π radians
d. $\frac{\pi}{2}$ radians
e. $\frac{\pi}{4}$ radians
f. $\frac{\pi}{3}$ radians
g. $\frac{\pi}{6}$ radians

8-77. Find your sine-calculator from Lesson 8.1.3. Use your new understanding of radians to convert the units on the θ axis from degrees to radians. Be prepared to share your conversion **strategies** with the class.

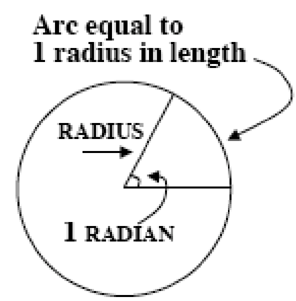


MATH NOTES

METHODS AND MEANINGS

Radians

A **radian** is defined as an angular measure such that an arc length of one radius on a circle of radius one produces an angle with measure one radian. It can also be thought of as the ratio of an arc length to the radius of the corresponding circle.



The circumference of a complete circle is $2\pi r$ units, so the corresponding radian measure is $\frac{2\pi r}{r} = 2\pi$. Thus, there are 2π radians in a complete circle.

Review & Preview

8-78. Your scientific or graphing calculator can function in both degrees and radians. See if you can figure out how to put your calculator in radian mode and then how to switch it back to degree mode. On most scientific calculators, a small "DEG" or "RAD" shows on the screen to let you know which mode you are in.

a. With your calculator in degree mode, find $\sin 60°$ and record your answer. Then switch to radian mode and find $\sin\frac{\pi}{3}$. Did you get the same answer? Explain why your answers should be the same or different.

b. Find $\sin\frac{\pi}{4}$. Which angles, measured in degrees, would have the same sine as $\sin\frac{\pi}{4}$?

8-79. Calculate each of the following values. Express your answers exactly and as decimal approximations.

a. $\sin(\frac{\pi}{4})$ b. $\sin(\frac{2\pi}{3})$

8-80. Colleen and Jolleen both used their calculators to find $\sin 30°$. Colleen got $\sin 30° = -0.9880316241$, but Jolleen got $\sin 30° = 0.5$. Is one of their calculators broken, or is something else going on? Why did they get different answers?

8-81. Recall the strategies developed in class for converting degrees to radians. How could you reverse that? Convert each of the following angle measures. Be sure to show all of your work.

a. π radians to degrees b. 3π radians to degrees

c. 30 degrees to radians d. $\frac{\pi}{4}$ radians to degrees

e. 225 degrees to radians f. $\frac{3\pi}{2}$ radians to degrees

8-82. Greg was working on his homework. He completed the square to change $y = 2x^2 - 6x + 2$ to graphing form and identify the vertex of the parabola. He did the work at right and identified the vertex to be $(\frac{3}{2}, -\frac{1}{4})$. When he got back to class and checked his answers, he discovered that his vertex was wrong, but he cannot find his mistake. Examine Greg's work and explain to him where the mistake occured. Then show him how to correct the mistake and state the vertex.

$y = 2x^2 - 6x + 2$

$y = 2(x^2 - 3x) + 2$

$y = 2(x^2 - 3x + \frac{9}{4}) + 2 - \frac{9}{4}$

$y = 2(x - \frac{3}{2})^2 - \frac{1}{4}$

8-83. Change each of the following equations to graphing form and then, without graphing, identify the vertex and axis of symmetry for each.

a. $y = 3x^2 - 18x + 26$ b. $y = 3x^2 - 4x - 11$

8-84. Solve each of the following equations for x.

a. $171 = 3(5^x)$ b. $171y = 3(x^5)$

8-85. Sketch a graph of $x^2 + y^2 = 100$.

a. Is it a function?

b. What are its domain and range?

c. Draw a central angle that measures $\frac{2\pi}{3}$ radians. If you remove this wedge of the circle, how much area remains?

8-86. Find the equation for the inverse of the function $f(x) = 2\sqrt{\frac{(x-3)}{4}} + 1$. Sketch the graph of both the original and the inverse.

8.1.6 What do I know about a unit circle?

Building a Unit Circle

In this lesson, you will further develop your understanding of the unit circle and how useful it can be. By the end of the lesson, you should be able to answer the questions below.

What can the unit circle help me understand about an angle?

What does my information about angles in the first quadrant tell me about angles in other quadrants?

8-87. There are some angles for which you know the *exact values* of sine and cosine. In other words, you can find the exact sine and cosine without using a calculator. Work with your team to find as many such angles (expressed in radians) as you can.

8-88. Now you will build a unit circle. Obtain the Lesson 8.1.6 Resource Page from your teacher. There are points shown at $\frac{\pi}{12}$, $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, $\frac{5\pi}{12}$, $\frac{7\pi}{12}$, $\frac{2\pi}{3}$, $\frac{3\pi}{4}$, $\frac{5\pi}{6}$, and $\frac{11\pi}{12}$ units along the circle, starting from the positive x-axis.

a. Find and label the exact coordinates, in (x, y) form, for three of the points shown in the *first quadrant.*

b. Mark *all* other points in the unit circle for which you can find *exact* coordinates. Not all of them are shown. Label each of these points with its angle of rotation (in radians) and its coordinates.

c. If you have not done so already, label each angle with its corresponding radian measure.

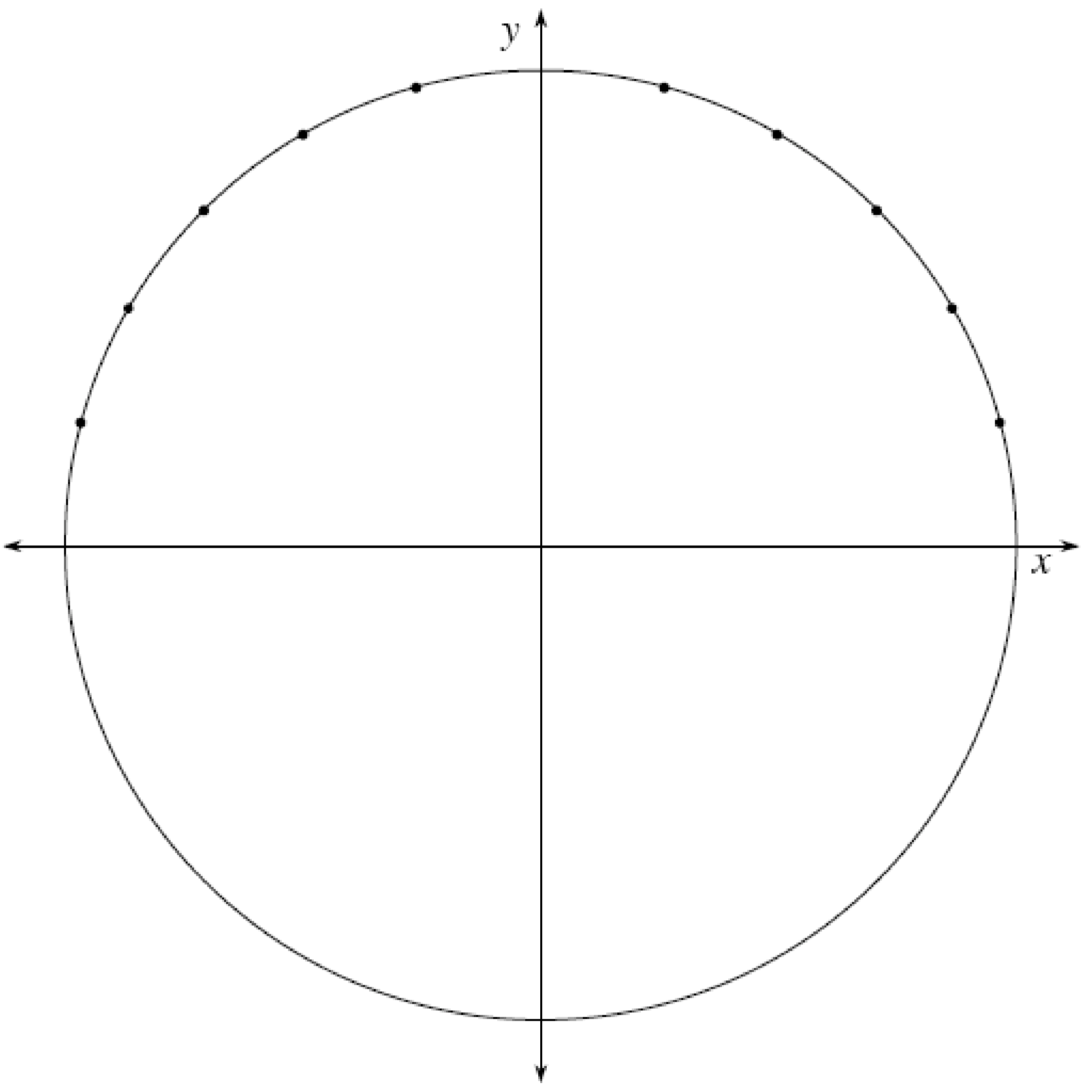
8-89. Draw a new unit circle, label a point that corresponds to a rotation of $\frac{\pi}{12}$, and put your calculator in radian mode.

a. What are the coordinates of this point, correct to two decimal places?

b. Use the information you found in part (a) to determine each of the following values: (Hint: Drawing each angle on the unit circle will be very helpful.)

i. $\sin(-\frac{\pi}{12})$ *ii.* $\cos\frac{13\pi}{12}$ *iii.* Challenge: $\cos\frac{7\pi}{12}$

Lesson 8.1.6 Resource Page



8-90. For angle α in the first quadrant, $\cos\alpha = \frac{8}{17}$. Use that information to find each of the following values without using a calculator. Be prepared to share your **strategies** with the class.

a. $\sin\alpha$

b. $\sin(\pi + \alpha)$

c. $\cos(2\pi - \alpha)$

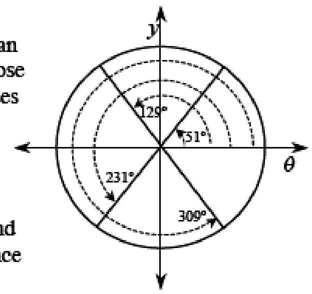
METHODS AND MEANINGS

MATH NOTES

Reference Angle

For every angle of rotation, there is an angle in the first quadrant ($0 \le \theta \le 90°$) whose cosine and sine have the same absolute values as the cosine and sine of the original angle. This first-quadrant angle is called the **reference angle**.

For example, the angles $51°$, $129°$, $231°$, and $309°$ (pictured at right) all share the reference angle of $51°$.



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Review & Preview

8-91. Calculate the value of each expression below. Each measure is given in radians.

a. $\sin(4)$ b. $\sin(\frac{4\pi}{3})$

8-92. Find the exact values of the angles that are solutions to the equation $\sin(\theta) = 0.5$. Express your solutions in radians.

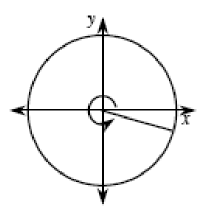
8-93. You have seen that you can calculate values of the sine function using right triangles formed by a radius of the unit circle. Values of θ that result in $30° - 60° - 90°$ or $45° - 45° - 90°$ triangles are used frequently on exercises and tests because their sines and cosines can be found exactly, without using a calculator. You should learn to recognize these values quickly and easily. The same is true for values of $\cos\theta$ and $\sin\theta$ that correspond to the *x*- and *y*-intercepts of the unit circle.

The central angles that correspond to these "special" values of *x* are 30°, 45°, 60°, 90°, 120°, 135°, 150°, 180°, 210°, 225°, 240°, 270°, 300°, 315°, and 330°. What these angles have in common is that they are all multiples of $30°$ or $45°$, and some of them are also multiples of $60°$ or $90°$.

Copy and complete a table like the one below for all special angles between 0° and 360°.

Degrees	0	30	45	60	90	120		
Radians	0	$\frac{\pi}{6}$						

8-94. Draw a picture of an angle that measures 6 radians.



a. Approximately how many degrees is this?

b. Using only your picture, estimate the sine of 6 radians.

8-95. Evaluate each expression without using a calculator or changing the form of the expression.

a. $\log(10)$ b. $\log(\sqrt{10})$

c. $\log(0)$ d. $10^{(2/3)\log(27)}$

8-96. What interest rate (compounded annually) would you need to earn in order to double your investment in 15 years?

8-97. Phana's garden is 2 meters wide and 5 meters long. She puts a walkway of uniform width around her garden. If the area of the walkway is 30 square meters, what are the outer dimensions of the walkway? Drawing a diagram will help you solve this problem.

8-98. Solve each of the following equations.

a. $2(x-1)^2 = 18$ b. $2^x + 3 = 10$

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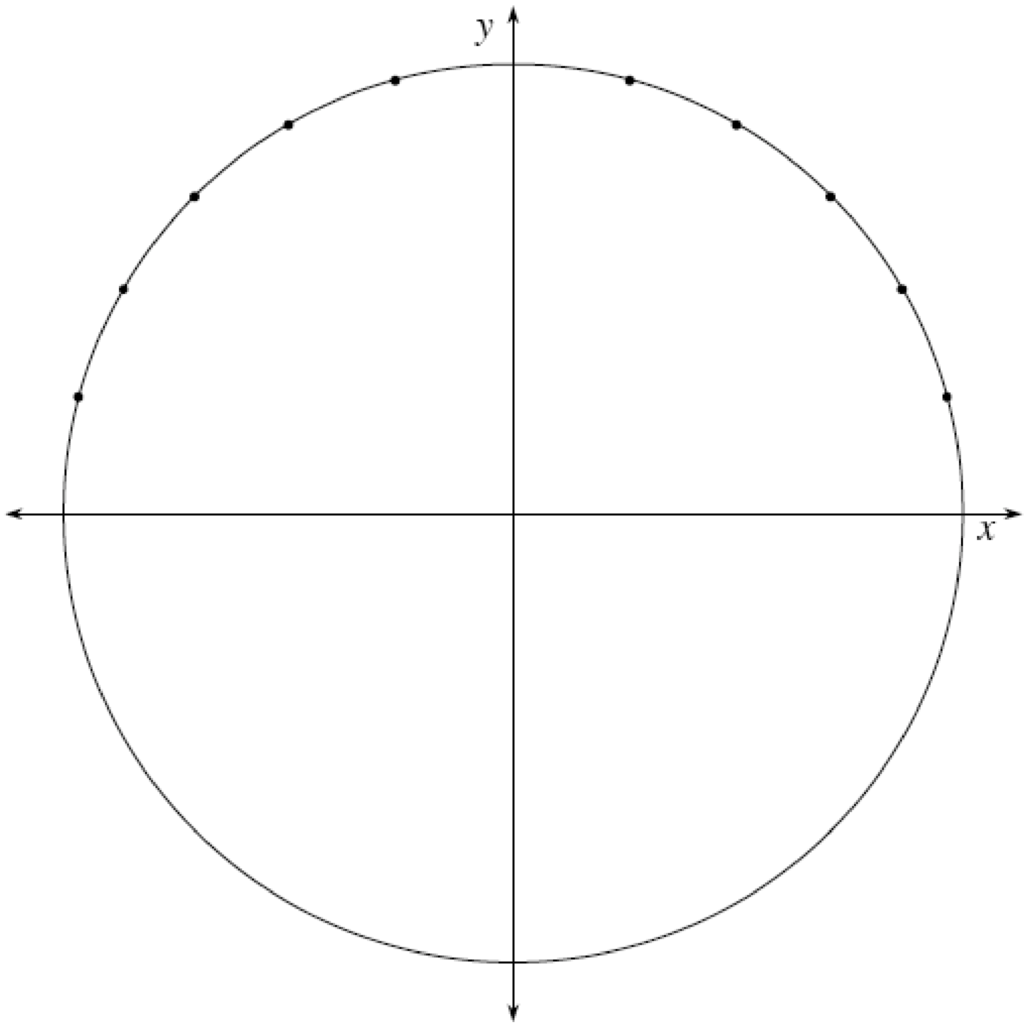
Lesson 8.1.6 Resource Page



8.1.7 What is tangent?

The Tangent Function

In the past several lessons, you have used your understanding of the sine and cosine ratios to develop and interpret the functions $s(\theta) = \sin\theta$ and $c(\theta) = \cos\theta$. In this lesson, you will expand your understanding by exploring the tangent ratio and graphing the function $t(\theta) = \tan\theta$.

8-99. Jamal was working on his homework when he had a brilliant realization. He was drawing a triangle in a unit circle to estimate the sine of $\frac{\pi}{10}$, when he realized that this triangle is the same kind of triangle that he draws when he wants to find the slope of a line.

a. How could you express the slope of the radius in terms of sine and cosine?

b. Is there any other way you can use a trigonometric ratio to represent the slope? Discuss this with your team.

8-100. THE TANGENT FUNCTION

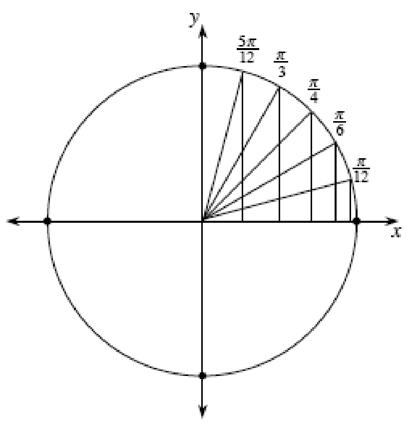
Obtain the Lesson 8.1.7 Resource Page from your teacher. Use your knowledge of sine, cosine, and tangent to create a graph of the tangent function. Conduct a full **investigation** of the tangent function. Be prepared to share summary statements with the class.

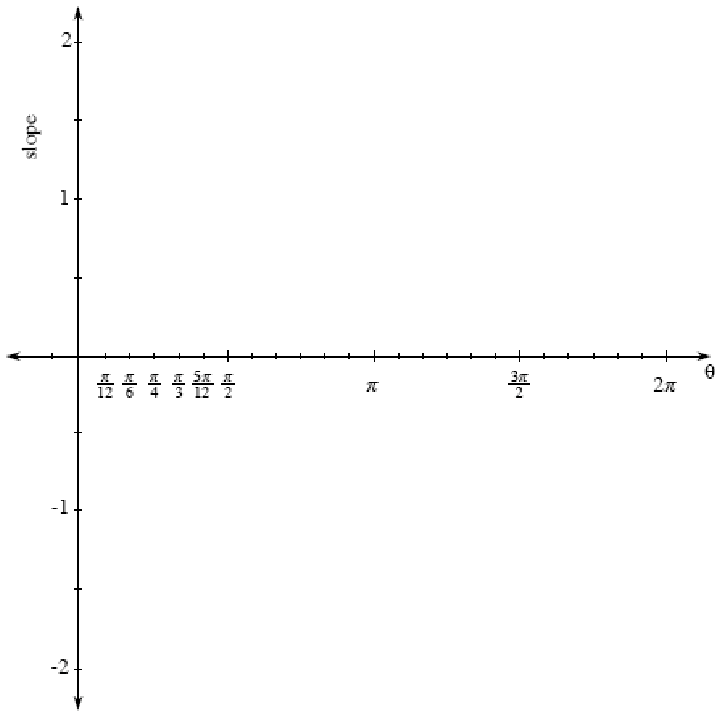
Discussion Points

Does every angle have a tangent value?

How is the tangent graph similar to or different from the sine and cosine graphs?

Why does the tangent graph have asymptotes?





Lesson 8.1.7 Resource Page

Further Guidance

8-101. For each triangle in the first quadrant of the unit circle on your resource page, label the sine and cosine.

a. Use your knowledge of tangent to complete a table like the one below. Start with the exact values for the sine and cosine.

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$ (exact)	$\tan\theta$ (approximate to nearest 0.01)
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$		
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$		

b. Plot the tangent values on the graph to the right of the unit circle.

c. Draw five new triangles that are congruent to the first five, but that are located in the second quadrant. Add values for these new angles to your table and your graph.

d. Continue this process by drawing triangles in the third and fourth quadrants. You should have a total of twenty triangles drawn and twenty angle values accounted for on your graph. If you have not done so already, add data to your table and points to your graph corresponding to the intercepts of the unit circle.

8-102. **Investigate** the tangent graph by analyzing the following questions:

a. Describe the domain and range of the tangent function.

b. Describe any special points or asymptotes.

c. Does it have symmetry? Describe any symmetry you see in the graph.

d. How is the graph of $t(\theta) = \tan\theta$ different from the graphs of $s(\theta) = \sin\theta$ and $c(\theta) = \cos\theta$?

Further Guidance section ends here.

8-103. Draw a new unit circle and label a point that corresponds to a rotation of $\frac{\pi}{6}$ radians.

a. What are the coordinates of this point? Use exact values.

b. Use this information to find each of the following values without a calculator. (Hint: Drawing each angle on the unit circle will be very helpful.)

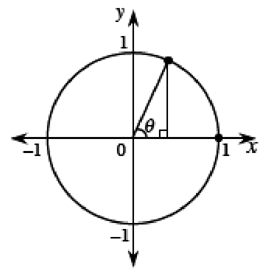
i: $\tan(\frac{7\pi}{6})$ *ii*: $\cos(\frac{13\pi}{6})$ *iii:* $\tan(\frac{2\pi}{3})$

MATH NOTES

METHODS AND MEANINGS

Sine, Cosine, and Tangent

For any real number θ, the **sine of θ**, denoted $\sin\theta$, is the y-coordinate of the point on the unit circle reached by a rotation of θ radians from **standard position** (counter-clockwise starting from the positive x-axis).



The **cosine of θ**, denoted $\cos\theta$, is the x-coordinate of the point on the unit circle reached by a rotation of θ radians from standard position.

The **tangent of θ**, denoted by $\tan\theta$, is the slope of the terminal ray of an angle (the radius) formed by a rotation of θ radians in standard position.

Review & Preview

8-104. What central angle, measured in degrees, corresponds to a distance around the unit circle of $\frac{7\pi}{3}$?

a. What other angles will take you to the same point on the circle?

b. Make a sketch of the unit circle showing the resulting right triangle.

c. Find $\sin(\frac{7\pi}{3})$, $\cos(\frac{7\pi}{3})$, and $\tan(\frac{7\pi}{3})$ exactly.

8-105. Evaluate each of the following trig expressions without using a calculator.

a. sin(180°) b. sin(360°) c. sin(–90°)

d. sin(510°) e. cos(90°) f. tan(–90°)

8-106. How do you convert from degrees to radians and from radians to degrees? Explain and **justify** your method completely.

8-107. Convert each of the following angle measures. Give exact answers

a. $\frac{7\pi}{6}$ radians to degrees

b. $\frac{5\pi}{3}$ radians to degrees

c. 45 degrees to radians

d. 100° to radians

e. 810° to radians

f. $\frac{7\pi}{2}$ radians to degrees

8-108. Simplify each of the following expressions, leaving only positive exponents in your answer.

a. $(x^3y^{-2})^{-4}$

b. $-3x^2(6xy - 2x^3y^2z)$

8-109. Sketch a graph of $f(x) = \frac{1}{2}(x+1)^3$. Then sketch its inverse and write the equation of the inverse.

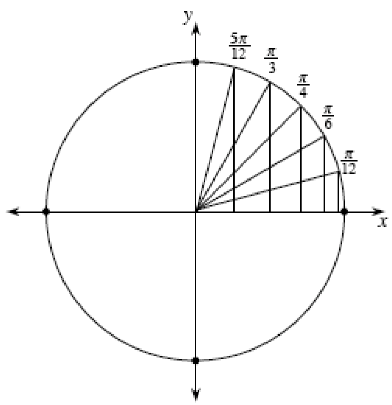
8-110. Rewrite $f(x) = 2x^2 - 16x + 34$ in graphing form.

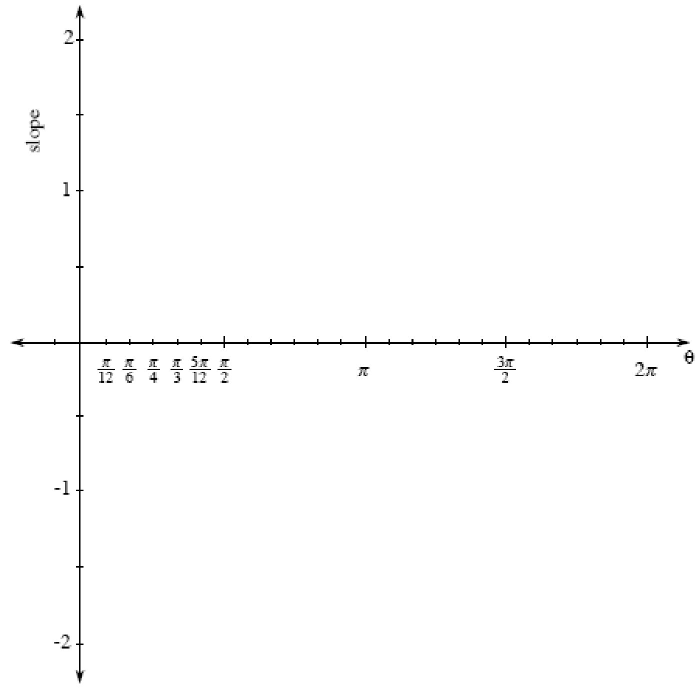
8-111. The temperature of a pizza after it has been delivered depends on how long it has been sitting on the family-room table.

a. Sketch a reasonable graph of this situation. Be sure to label the axes.

b. Should your graph have an asymptote? Why or why not?

8-112. Solve for x: $\frac{1}{ax} + \frac{1}{b} = \frac{1}{x}$





Lesson 8.1.7 Resource Page

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8.2.1 How can I transform a sine graph?

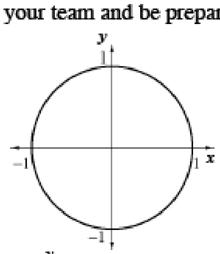
Transformations of $y = \sin x$

In Chapter 4, you developed expertise in **investigating** functions and transforming parent graphs. In this section, you will **investigate** families of cyclic functions and their transformations. By the end of this section, you will be able to graph any sine or cosine equation and write the equation of any sine or cosine graph.

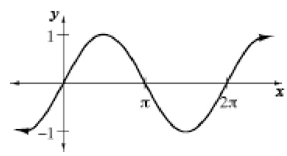
8-113. As you have seen with many functions in this and other courses, x is generally used to represent an input and y is used to represent the corresponding output. By this convention, sinusoidal functions should be written $y = \sin x$, $y = \cos x$, and $y = \tan x$. But beware! Something funny is happening.

With your team, examine the unit circle and the three graphs below. What do x and y represent in the unit circle? What do they represent in each of the graphs? Discuss this with your team and be prepared to share your ideas with the class.

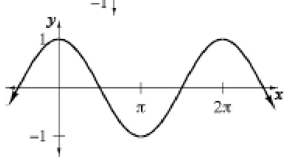
a.



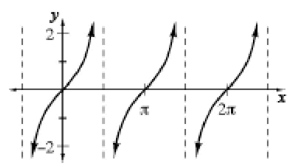
b.



c.



d.



8-114. With your team, you will apply your knowledge about transforming graphs of functions to transform the graphs of $y = \sin x$ and $y = \cos x$ and find their general equations.

Your task: As a team, **investigate** $y = \sin x$ and $y = \cos x$ completely. You should make graphs, find the domain and range, and label any important points or asymptotes. Then make a sketch and write an equation to demonstrate each transformation of the sine or cosine function you can find. Finally, find a general equation for a sine and a cosine function. Be prepared to share your summary statements with the class.

Discussion Points

What can we change in a cyclic graph?

Which points are important to label?

How can we apply the transformations we use with other functions?

Are there any new transformations that are special to the sine function?

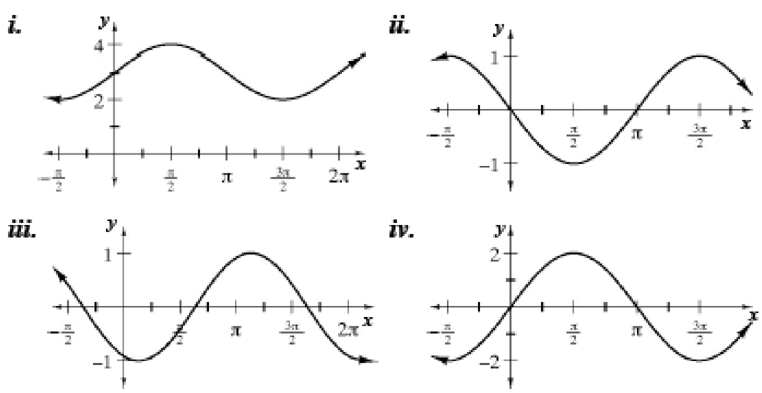
Further Guidance

8-115. Sketch a graph of at least one cycle of $y = \sin x$. Label the intercepts. Then work with your team to complete parts (a) through (c) below.

a. Write an equation for each part below and sketch a graph of a function that has a parent graph of $y = \sin x$, but is:

i. Shifted 3 units up. *ii.* Reflected across the x-axis.

iii. Shifted 2 units to the right. *iv.* Vertically stretched.



b. Which points are most important to label in a periodic function? Why?

c. Write a general equation for the family of functions with a parent graph of $y = \sin x$.

Further Guidance section ends here.

Review & Preview

8-116. Imagine the graph $y = \sin(x)$ shifted up one unit.

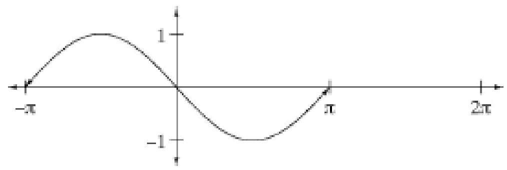
a. Sketch what it would look like.

b. What do you have to change in the equation $y = \sin x$ to move the graph *up* one unit? Write the new equation.

c. What are the intercepts of your new equation? Label them with their coordinates on the graph.

d. When you listed intercepts in part (c), did you list more than one x-intercept? Should you have?

8-117. The graph at right was made by shifting the first cycle of $y = \sin x$ to the left.



a. How many units to the left was it shifted?

b. Figure out how to change the equation of $y = \sin x$ so that the graph of the new equation will look like the one in part (a). If you do not have a graphing calculator at home, sketch the graph and check your answer when you get to class.

8-118. Which of the situations below (if any) is best modeled by a cyclic function? Explain your reasoning.

a. The number of students in each year's graduating class.

b. Your hunger level throughout the day.

c. The high-tide level at a point along the coast.

8-119. The CPM Amusement Park has decided to imitate *The Screamer* but wants to make it even better. Their ride will consist of a circular track with a radius of 100 feet, and the center of the circle will be 50 feet under ground. Passengers will board at the highest point, so they will begin with a blood-curdling drop. Write a function that relates the angle traveled *from the starting point* to the height of the rider above or below the ground.

8-120. Should $y = \sin x$ and $y = \cos x$ both be parent graphs, or is one the parent of the other? Give reasons for your decision.

8-121. Evaluate each of the following expressions exactly.

a. $\tan\frac{2\pi}{3}$

b. $\tan\frac{7\pi}{6}$

8-122. David Longshot is known for his long golf drives. Today he hit the ball 250 yards. He estimated that the ball reached a maximum height of 15 yards. Find a quadratic equation that would model the path of the golf ball.

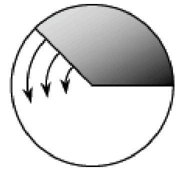
8.2.2 What is missing?

One More Parameter for a Cyclic Function

In this lesson, you will study one more transformation that is unique to cyclic functions. You will also extend your understanding of these functions to include those with input values that do not correspond to angles.

8-123. Does the general equation $y = a\sin(x-h)+k$ allow for every possible transformation of the graph of $y = \sin x$? Are there any transformations possible *other than* the ones produced by varying values of a, h, and k? Look back at the graphs you made for the swinging bag of blood in the first lesson of this chapter. Discuss this with your team and be prepared to share your conjectures with the class.

8-124. THE RADAR SCREEN



Brianna is an air traffic controller. Every day she watches the radar line (like a radius of a circle) go around her screen time after time. On one particularly slow travel day, Brianna noticed that it takes 2 seconds for the radar line to travel through an angle of $\frac{\pi}{6}$ radians. She decided to make a graph in which the *input* is time and the *output* is the distance from the outward end of the radar line to the horizontal axis.

Your task: Following the input and output specifications above, make a table and graph for Brianna's radar.

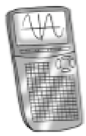
Discussion Points

How can we calculate the outputs?

How is this graph different from other similar graphs we have made?

How long does it take to complete one full cycle on the radar screen? How can we see that on the graph?

8-125. Now that you have seen that it is possible to have a sine graph with a cycle length that is not 2π, work with your team to make conjectures about how you could change your general equation to allow for this new transformation.



a. In the general equation $y = a\sin(x-h)+k$, the quantities a, h, and k are called **parameters**. Where could a new parameter fit into the equation?

b. Use your graphing calculator to test the result of putting this new parameter into your general equation. Once you have found the place for the new parameter, **investigate** how it works. What happens when it gets larger? What happens when it gets smaller?

c. Write a general equation for a sine function that includes the new parameter you discovered.

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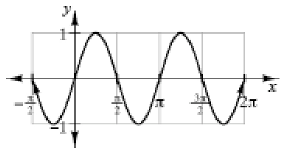
8-126. Which of the following have a period of 2π ? Which do not? How can you tell? If the period is not 2π , what is it?

a.



b. A pendulum takes 3 seconds to complete one cycle.

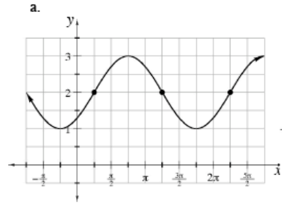
c. $y = \sin\theta$

d. A radar line takes 1 second to travel through 1 radian.

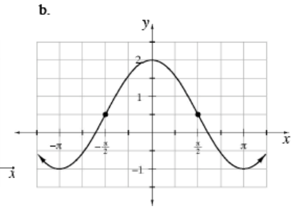
Review & Preview

8-127. Find an equation for each graph below.

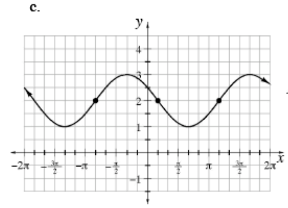
a.



b.



c.



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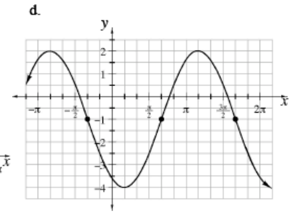


8-128. Claudia graphed $y = \cos\theta$ and $y = \cos(\theta + 360°)$ on the same set of axes. She did not see any difference in their graphs at all. Why not?

8-129. This is a checkpoint for solving equations and inequalities involving absolute value.

Solve each absolute value equation or inequality below.

a. $2|2x+3| = 10$

b. $|3x-5| > 13$

c. $-|x+3| < 10$

d. Check your answers by referring to the Checkpoint 16 materials located at the back of your book.

If you needed help to solve these equations and inequalities correctly, then you need more practice solving equations and inequalities with absolute value. Review the Checkpoint 16 materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to solve equations and inequalities such as these quickly and accurately.

8-130. Find the *x*- and *y*-intercepts of the graphs of each of the following equations.

a. $y = 2x^3 - 10x^2 - x$

b. $y + 2 = \log_3(x-1)$

8-131. The average cost of movie tickets is \$9.50. If the cost is increasing 4% per year, in how many years will the cost double?

8-132. Change each equation to graphing form. For each equation, find the domain and range and determine if it is a function.

a. $y = -2x^2 - x + 13$

b. $y = -3x^2 - 6x + 12$

8-133. Too Tall Thomas has put Rodney's book bag on the snack-shack roof. Rodney goes to borrow a ladder from the school custodian. The tallest ladder available is 10 feet long and the roof is 9 feet from the ground. Rodney places the ladder's tip at the edge of the roof. The ladder is unsafe if the angle it makes with the ground is more than 60°. Is this a safe situation? **Justify** your conclusion.

8-134. Deniz's computer is infected with a virus that will erase information from her hard drive. It will erase information slowly at first, but as time goes on, the rate at which information is erased will increase. In *t* minutes after the virus starts erasing information, $5{,}000{,}000(\frac{1}{2})^t$ bytes of information remain on the hard drive.

a. Before the virus starts erasing, how many bytes of information are on Deniz's hard drive?

b. After how many minutes will there be 1000 bytes of information left on the drive?

c. When will the hard drive be completely erased?

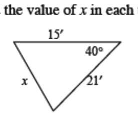
8-135. Graph $f(x) = |x-6| - 4$.

a. Explain how you can graph this without making an $x \to y$ table, but using parent graphs.

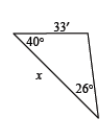
b. Graph $g(x) = ||x-6| - 4|$. Explain how you can graph $g(x)$ without making an $x \to y$ table by using your earlier graph.

8-136. Find the value of *x* in each triangle.

a.



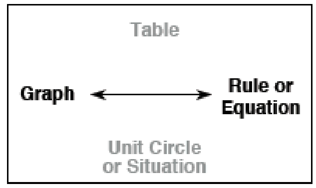
b.



8.2.3 What is the period of a function?

Period of a Cyclic Function

In Lesson 8.2.2, you found a place for a new parameter in the general form of a trigonometric equation and discovered that it must have something to do with the period. By the end of this lesson, you will have the tools you need to find the equation for any sine or cosine graph and will be able to graph any sine or cosine equation. In other words, you will learn the equation ↔ graph connection. The following questions can help your team stay focused on the purpose of this lesson.



How can we write the equation for any sine or cosine graph?

How can we graph any sine or cosine function?

8-137. Find the period for each of the following situations:

a. The input is the angle θ in the unit circle, and the output is the cosine of θ.

b. The input is time, and the output is the average daily temperature in New York.

c. The input is the distance Nurse Nina has traveled along the hallway, and the output is the distance of bloody drips from the center-line of the hallway.

8-138. Put your graphing calculator in radian mode. Set the domain and range of the viewing window so that you would see just one complete cycle of $y = \sin x$. What is the domain for one cycle? Range?

a. Graph $y = \sin x$, $y = \sin(0.5x)$, $y = \sin(2x)$, $y = \sin(3x)$, and $y = \sin(5x)$. Make a sketch and answer the following questions for each equation.

i. How many cycles of each graph appear on the screen?

ii. What is the amplitude (height) of each graph?

iii. What is the period of each graph?

iv. Is each equation a function?

b. Make a conjecture about the graph of $y = \sin(bx)$ with respect to each of the questions (*i*), (*ii*), (*iii*), and (*iv*) above. If you cannot make a conjecture yet, try more examples.

c. Create at least three of your own examples to check your conjectures. Be sure to include sketches of your graphs.

d. What is the relationship between the period of a sine graph and the value of b in its equation?

8-139. Take the graph you made by swinging a pendulum in Lesson 8.1.1. Decide where to draw x- and y-axes and find the equation of your graph. Is there more than one possible equation? Be prepared to share your **strategies** with the class.

8-140. *Without* using a graphing calculator, describe each of the following functions by stating the amplitude, period, and vertical and horizontal shifts. Then sketch the graph of each function. *After* you have completed each graph, check your sketch with the graphing calculator and correct and explain any errors.

a. $y = \sin 2(x - \frac{\pi}{6})$

b. $y = 3 + \sin(\frac{1}{3}x)$

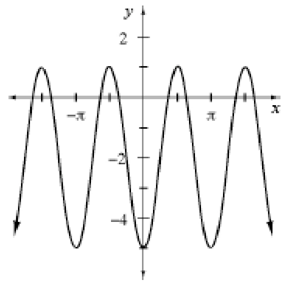
c. $y = 3\sin(4x)$

d. $y = \sin \frac{1}{2}(x + 1)$

e. $y = -\sin 3(x - \frac{\pi}{3})$

f. $y = -1 + \sin(2x - \frac{\pi}{2})$

8-141. Farah and Thu were working on writing the equation of a sine function for the graph at right. They figured out that the amplitude is 3, the horizontal shift is $\frac{\pi}{4}$ and the vertical shift is –2. They can see that the period is π, but they disagree on the equation. Farah has written $f(x) = 3\sin 2(x - \frac{\pi}{4}) - 2$ and Thu has written $f(x) = 3\sin(2x - \frac{\pi}{4}) - 2$.



a. Whose equation is correct? How can you be sure?

b. Graph the incorrect equation and explain how it is different from the original graph.

8-142. Look back at the general equation you wrote for the family of sine functions in problem 8-114. Now that you have figured out how period affects the equation, work with your team to add a new parameter (call it *b*) that allows your general equation to account for any transformation of the sine function, including changes in the length of each cycle. Be prepared to share your general equation with the class.

Review & Preview

8-143. Based on your explorations in class, complete parts (a) through (c) below.

a. Describe what the graph of $y=3\sin(\frac{1}{2}x)$ will look like compared to the graph of $y=\sin x$.

b. Sketch both graphs on the same set of axes.

c. Explain the similarities and differences between the two graphs.

8-144. What is the period of $y=\sin(2\pi x)$? How do you know?

8-145. Sketch the graph of each equation below.

a. $y=\sin(2\pi x)$ b. $y=3\sin(\pi x)$

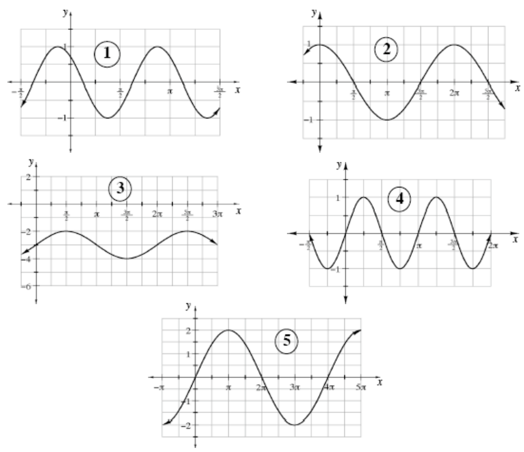
c. $y=2\sin(2\pi x)+1$

8-146. Match each equation with the appropriate graph. Do this without using a graphing calculator.

a. $y=\sin(x+\frac{\pi}{2})$ b. $y=\sin(2x)$

c. $y=2\sin(\frac{x}{2})$ d. $y=\sin(x)-3$

e. $y=-\sin[2(x-\frac{\pi}{8})]$



8-147. Ceirin's teacher promised a quiz for the next day, so Ceirin called Adel to review what they had done in class. *"Suppose I have* $y=\sin 2x$*,"* said Ceirin, *"what will its graph look like?"*

"It will be horizontally compressed by a factor of 2," replied Adel, *"so the period must be* π*."*

"Okay, now let's say I want to shift it one unit to the right. Do I just subtract 1 from x, like always?"

"I think so," said Adel, *"but let's check on the graphing calculator."* They proceeded to check on their calculators. After a few moments they both spoke at the same time.

"Rats," said Ceirin, *"it isn't right."*

"Cool," said Adel, *"it works."*

When they arrived at school the next morning, they compared the equations they had put in their graphing calculators while they talked on the phone. One had $y=\sin 2x-1$, while the other had $y=\sin 2(x-1)$.

Which equation was correct? Did they both subtract 1 from x? Explain. Describe the rule for shifting a graph one unit to the right in a way that avoids this confusion.

8-148. George was solving the equation $(2x-1)(x+3)=4$ and he got the solutions $x=\frac{1}{2}$ and $x=-3$. Jeffrey came along and said, *"You made a big mistake! You set each factor equal to zero, but it's not equal to zero, it's equal to 4. So you have to set each factor equal to 4 and then solve."* Who is correct? Show George and Jeffrey how to solve this equation. To be sure that you are correct, check your solutions.

8-149. Complete the square to change $3x^2+6x+3y^2-9y<36$ to graphing form. Identify key points. Find the domain and range. Sketch the graph.

8-150. Simplify each expression without using a calculator.

a. $25^{-1/2}$ b. $(\frac{1}{27})^{-1/3}$

]c. $9^{3/2}$ d. $16^{-3/4}$

8-151. Consider the equation $f(x)=3(x+4)^2-8$.

a. Find an equation of a function $g(x)$ such that $f(x)$ and $g(x)$ intersect in only one point.

b. Find an equation of a function $h(x)$ such that $f(x)$ and $h(x)$ intersect in no points.

8.2.4 What are the connections?

Graph ↔ Equation

In the past few lessons, you have been developing the understanding necessary to graph a cyclic equation without making a table and to write an equation from a cyclic graph. In today's lesson, you will strengthen your understanding of the connections between a cyclic equation and its graph. By the end of this lesson, you will be able to answer the following questions:

Table

Graph ↔ **Rule or Equation**

Unit Circle or Situation

Does it matter if we use sine or cosine?

What do we need to know to make a complete graph or write an equation?

8-152. What do you need to know about the sine or cosine functions to be able to graph them or write their equations? Talk with your team and write a list of all of the attributes of a sine or cosine function that you need to know in order to write an equation and graph it.

8-153. CREATE-A-CURVE

Split your team into partners. With your partner, you will create your own sine or cosine function, write its equation, and draw its graph. Be sure to keep your equation and graph a secret! Start by choosing whether you will work with a sine or a cosine function.

a. Half the distance from the highest point to the lowest point is called the **amplitude**. You can also think of amplitude as the vertical stretch. What is the amplitude of your function?

b. How far to the left or right of the y-axis will your graph begin? In other words, what will be the **horizontal shift** of your function?

c. How much above or below the x-axis will the center of your graph be? In other words, what will be the **vertical shift** of your function?

d. What will the **period** of your function be?

e. What will the **orientation** of your graph in relation to $y = \sin x$ or $y = \cos x$ be? Is it the same or is it flipped?

f. Now that you have decided on all of the attributes for your function, write its equation.

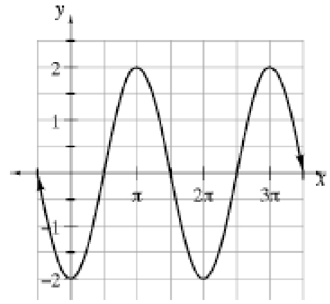
8-154. Copy the equation for your curve from problem 8-153 on a clean sheet of paper. Trade papers with another pair of students.

a. Sketch a graph of the equation you received from the other pair of students.

b. When you are finished with your graph, give it back to the other pair so that they can check the accuracy of your graph.

8-155. When you look at a graph and prepare to write an equation for it, does it matter if you choose sine or cosine? Which will work best?

With your team, find *at least four* different equations for the graph at right. Be prepared to share your equations with the class.



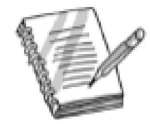
a. Does it matter if you choose sine or cosine?

b. Which of your equations do you prefer? Why?

8-156. Sarita was watching her little sister bounce on a trampoline and she decided to take some data, so she started her stopwatch. Half a second later, her sister reached the highest point, 15 feet above the ground! When the stopwatch read 1.4 seconds, Sarita's sister was at the lowest point, stretching the trampoline down to just one foot above the ground. Find a cyclic equation that models the height of Sarita's sister over time if she continues to bounce in the same way.

April 02, 2011

8-157. LEARNING LOG



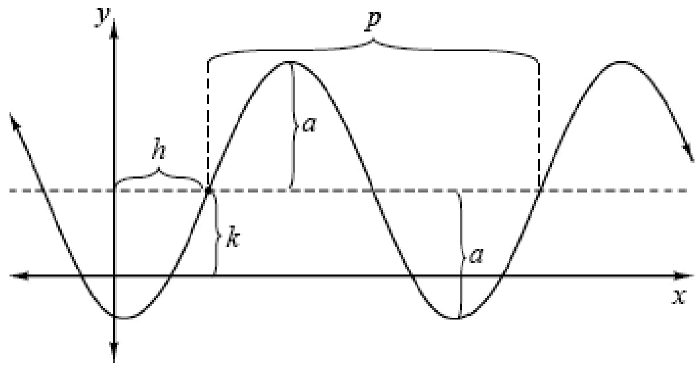
In your Learning Log, write your ideas about the target questions for this lesson: *Does it matter if I use sine or cosine? What do I need to know to make a complete graph or write an equation?* Title this entry "Cyclic Equations and Graphs" and label it with today's date.

METHODS AND MEANINGS

MATH NOTES

General Equation for Sine Functions

The **general equation** for the **sine function** is $y = a\sin[b(x-h)]+k$.



The **amplitude** (half of the distance between the highest and the lowest points) is ***a***.

The **period** is the length of one cycle. It is labeled ***p*** on the graph.

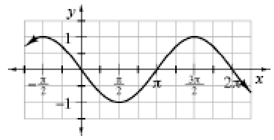
The number of cycles in 2π is ***b***.

The **horizontal shift** is ***h***.

The **vertical shift** is ***k***.

Review & Preview

8-158. Susan knew how to shift $y = \sin x$ to get the graph at right, but she wondered if it would be possible to get the same graph by shifting $y = \cos x$.



a. Is it possible to write a cosine function for this graph?

b. If you think it is possible, find an equation that does it. If you think it is impossible, explain why.

c. Adlai said, *"I can get that graph* without *shifting to the right or left."* What equation did he write?

8-159. In the function $y = 4\sin(6x)$, how many cycles of sine are there from 0 to 2π? How long is each cycle (i.e., what is the period)?

8-160. Write the equation of a cyclic function that has an amplitude of 7 and a period of 8π. Sketch its graph.

8-161. Sketch a graph of each of the following trigonometric functions.

a. $y = \sin(0.5(x - \pi))$ b. $y = 10\sin(3x) - 2$

c. $y = 5\cos(x + \frac{\pi}{4})$ d. $y = \cos(2(x - \frac{\pi}{4}))$

8-162. Find the exact value for each of the following trig expressions. For parts (g) and (h), assume that $0 \le \theta \le 2\pi$.

a. $\cos(\frac{3\pi}{4}) =$ b. $\tan(\frac{4\pi}{3}) =$ c. $\sin(\frac{11\pi}{6}) =$ d. $\sin(\frac{3\pi}{4}) =$

e. $\tan(\frac{5\pi}{4}) =$ f. $\tan(\frac{17\pi}{6}) =$ g. $\tan(\theta) = 1$ h. $\tan(\theta) = -1$

8-163. Use the Zero Product Property to solve each equation in parts (a) and (b) below.

a. $x(2x+1)(3x-5) = 0$ b. $(x-3)(x-2) = 12$

c. Write an equation and show how you can use the Zero Product Property to solve it.

8-164. Find a quadratic equation whose graph has each of the following characteristics:

a. No x-intercepts and a negative y-intercept.

b. One x-intercept and a positive y-intercept.

c. Two x-intercepts and a negative y-intercept.

8-165. A two-bedroom house in Seattle was worth \$400,000 in 2005. If it appreciates at a rate of 3.5% each year:

a. How much will it be worth in 2015?

b. When will it be worth \$800,000?

c. In Jacksonville, houses are depreciating at 2% per year. If a house is worth \$200,000 now, how much value will it have lost in 10 years?

April 02, 2011

CL 8-166. Describe how you can tell the difference between the graphs of $y = \sin x$ and $y = \cos x$. Be sure to **justify** your ideas.

CL 8-167. Convert the following angles to radians.

a. 225° b. 75° c. -15° d. 330°

CL 8-168. Sketch each of the following angles in its own unit circle.

a. An angle that has a positive cosine and a negative sine.
b. All angles that have a sine of 0.5.
c. An angle that measures $\frac{4\pi}{3}$ radians. Find its exact sine.
d. An angle with a negative cosine and a positive tangent.

CL 8-169. Without using a calculator, give the exact value of each expression.

a. $\sin 60^\circ$ b. $\cos 180^\circ$ c. $\tan 225^\circ$
d. $\sin \frac{\pi}{4}$ e. $\cos \frac{2\pi}{3}$ f. $\tan \frac{3\pi}{2}$

CL 8-170. If an angle between 0 and 2π radians has a sine of -0.5, what is its cosine? How do you know?

a. No x-intercepts and a negative y-intercept.
b. One x-intercept and a positive y-intercept.
c. Two x-intercepts and a negative y-intercept.

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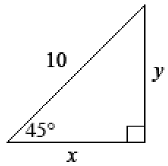
d. $\sin \frac{\pi}{4}$ e. $\cos \frac{2\pi}{3}$ f. $\tan \frac{3\pi}{2}$

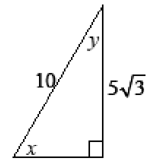
CL 8-170. If an angle between 0 and 2π radians has a sine of -0.5, what is its cosine? How do you know?

CL 8-171. Find the exact values of x and y in the drawings below.

a.



b.

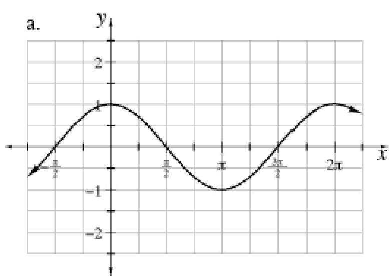


CL 8-172. For each equation determine the amplitude, period, locator point and sketch part of the graph.

a. $y = 3\cos(2x)$ b. $y = \tan(x - \frac{\pi}{2})$

CL 8-173. Write an equation for each of the following graphs. If you have a graphing calculator, use it to check your equation (be sure to set your window to match the picture).

a.



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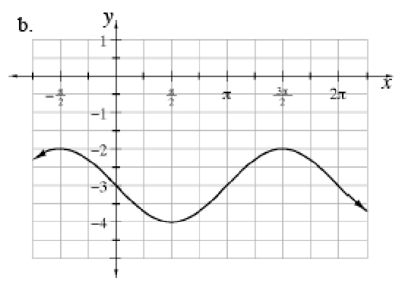
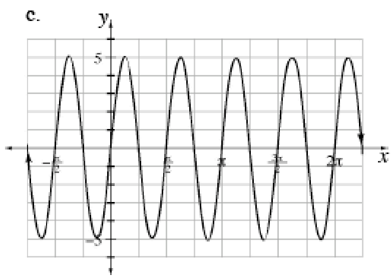
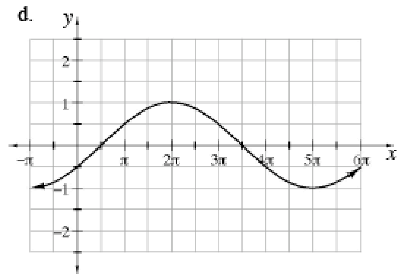


c.



d.



CL 8-174. Rewrite each equation below in graphing form and sketch its graph. Then state the domain and range and whether or not it is a function.

a. $y = 3x^2 + 30x - 2$ b. $x^2 + y^2 - 6x + 4y + 4 = 0$

CL 8-175. Solve each equation to the nearest thousandth.

a. $2 \cdot 3^x = 40.8$ b. $3x^4 = 27$

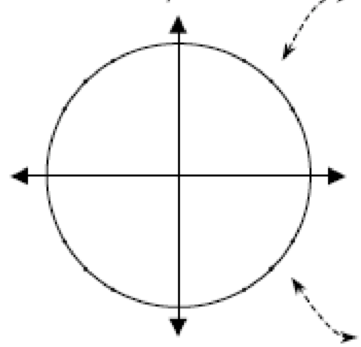
c. $\log_5(2x+1) = 3$ d. $\log(x) + \log(2x) = 5$

CL 8-176. Find an equation for an exponential function that passes through the points (1, 22) and (3, 20.125) and has a horizontal asymptote at $y = 20$.

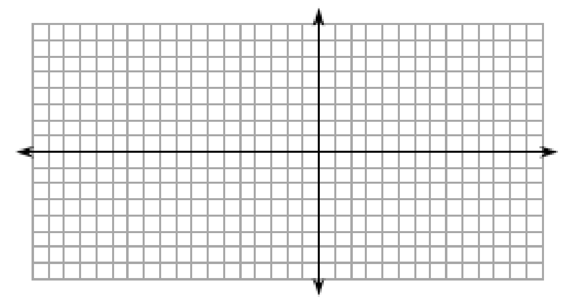
CL 8-177. Check your answers using the table at the end of this section. Which problems do you feel confident about? Which problems were hard? Have you worked on problems like these in previous chapters math classes? Sort the problems into three groups: the ones you are confident you can do, the ones you need more practice with, and the ones you need further help to understand.

Chapter 8 Closure Resource Page: Trigonometric Functions and the Unit Circle GO

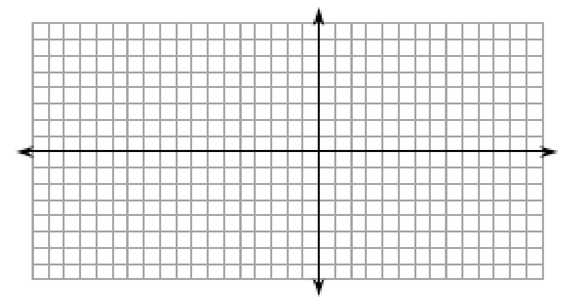
How does the unit circle connect to the graph, table and equation for each of the trigonometric functions you have learned about in this chapter? Use colors, arrows and other tools to show as many connections as you can between the unit circle and the other representations.



Multiple Representations of $f(x) = \sin x$:



Multiple Representations of $g(x) = \cos x$:



Multiple Representations of $h(x) = \tan x$: