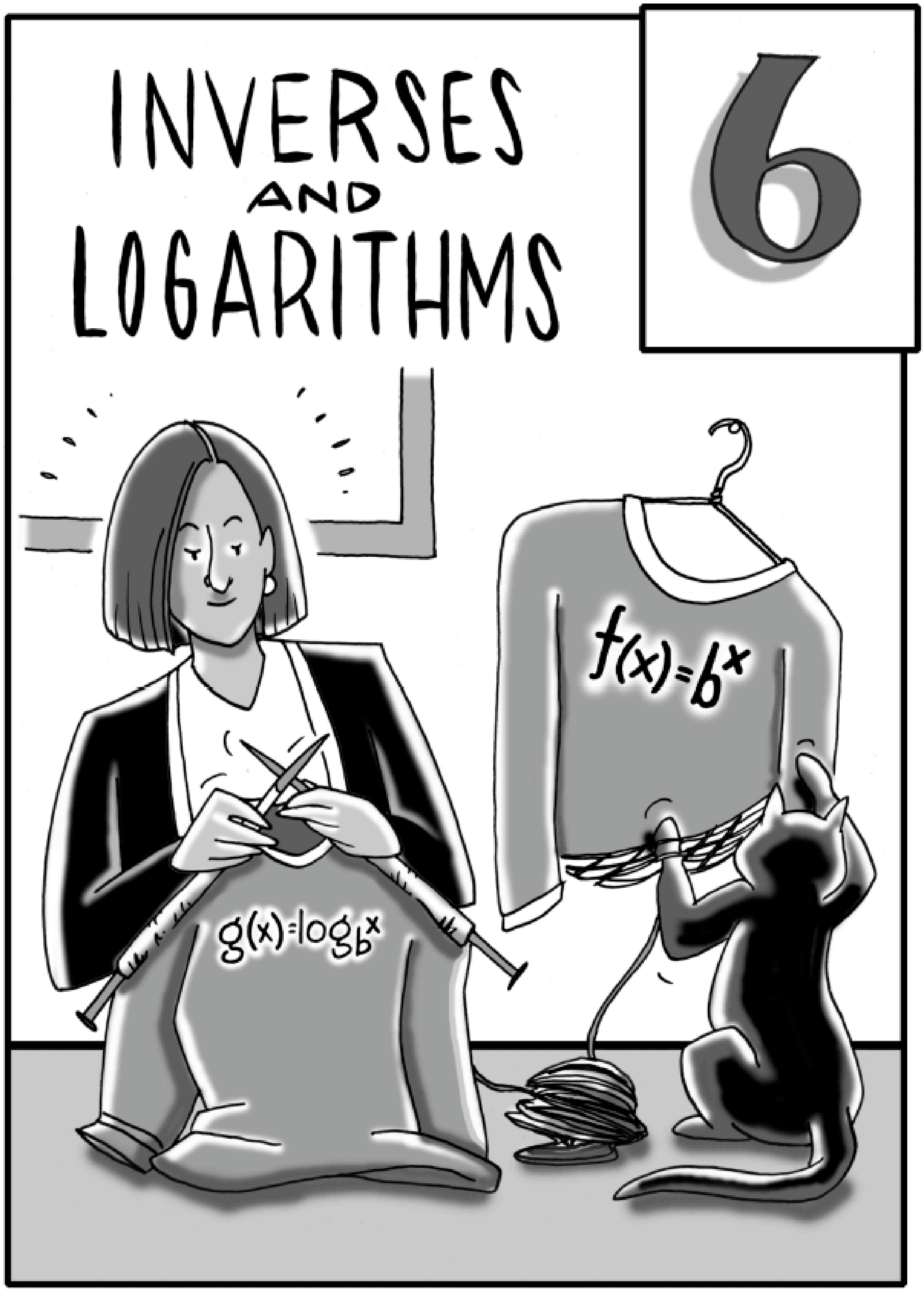
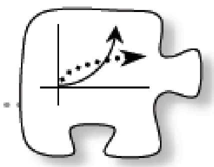

INVERSES
AND
LOGARITHMS
6
f(x)=b^x
g(x)=log_b x

Chapter 6 Teacher Guide

Section	Lesson	Days	Lesson Title	Materials	Homework
6.1	6.1.1	1	Undo Rules	• Lesson 6.1.1 Res. Pg. transparency	6-7 to 6-15
	6.1.2	2	Using a Graph to Find the Inverse	• Straight edge • Pencils or crayons • Lesson 6.1.2 Res. Pg. • Lesson 1.1.2A or E Res. Pg. (optional)	6-26 to 6-32 and 6-33 to 6-37
	6.1.3	1	Finding Inverses and Justifying Algebraically	• Unlined paper • Markers or colored pencils	6-44 to 6-53
6.2	6.2.1	1	Finding the Inverse of an Exponential Function	None	6-59 to 6-66
	6.2.2	1	Defining the Inverse of an Exponential Function	None	6-72 to 6-80
	6.2.3	1	Investigating the Family of Logarithmic Functions	• Transparencies • Overhead pens • Computer and projector • Dynamic Tool: *Exponential and Logarithmic Functions*	6-84 to 6-92
	6.2.4	1	Transformations of Logarithmic Functions	None	6-96 to 6-105
	6.2.5 (optional)	2	Investigating Compositions of Functions	None	6-113 to 6-120
Chapter Closure		Varied Format Options			

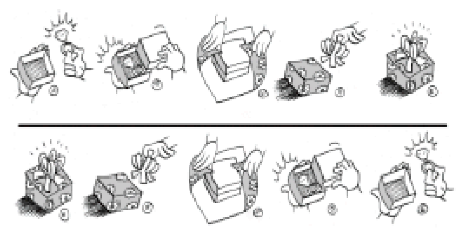
Total: **10 days plus optional closure time**

6.1.1 How can I "undo" a function?



"Undo" Rules

Have you ever heard the expression, "She knows it forward and backward," to describe someone who understands an idea deeply? Often, being able to **reverse** a process is a way to show how thoroughly you understand it. Today you will **reverse** mathematical processes, including functions. As you work today, keep these questions in mind:



How can I "undo" it?

How can I **justify** each step?

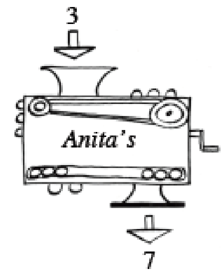
6-1. GUESS MY NUMBER

Today you will play the "Guess My Number" game. Your teacher will think of a number and tell you some information about that number. You will try to figure out what your teacher's number is. (You can use your calculator or paper if it helps.) When you think you know the number, sit silently and do not tell anyone! Be sure to give others a chance to figure it out!

For example your teacher might say: *"When I add 4 to my number and then multiply the sum by 10, I get –70. What is my number?"*

Your task will be to find the number and explain your reasoning.

6-2. A picture of Anita's function machine is shown at right. When she put 3 into the machine, 7 came out. When she put in 4, 9 came out, and when she put in –3, –5 came out.



a. Make a table to organize the inputs and outputs from Anita's function machine. Explain in words what this machine is doing to the input to generate an output.

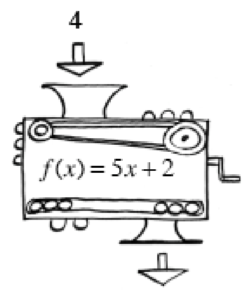
b. Anita's function machine suddenly starts working backwards: it is pulling outputs back up into the machine, **reversing** the machine's process, and returning the original input. If 7 is being pulled back into this machine, what value do you think will come out of the top? Anita sets up her new backwards function machine and enters the other ouputs. What would you expect to come out the top if 9 is entered? If –5 is entered? Explain.

6-2. *Problem continued from previous page.*

c. Record the inputs and outputs of the backwards function machine in a table. Record the numbers going in as x, and the numbers coming out as y. Explain in words what Anita's backwards function machine is doing.

d. Write rules for Anita's original function machine and for her backwards machine. How are the two rules related?

6-3. The function machine at right follows the rule $f(x) = 5x + 2$.



a. If the crank is turned backwards, what number should be pulled up into the machine in order to have a 4 come out of the top?

b. Keiko wants to build a new machine that will **undo** what $f(x)$ does to an input. What must Keiko's machine do to 17 to undo it and return a value of 3? Write your undo rule in function notation and call it $g(x)$.

c. Choose a value for x. Then find a **strategy** to show that your rule, $g(x)$, undoes the effects of the function machine $f(x)$.

6-4. Find the undo rules for each of the functions below. Use function notation and give the undo rule a name different from the original function's. **Justify** that each undo rule works for its function.

a. $f(x) = 3x - 6$

b. $h(x) = x^3 - 5$

c. $p(x) = 2(x+3)^3$

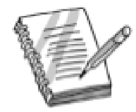
d. $t(x) = \frac{10(x-4)}{3}$

6-5. Each team member should choose one function and its undo rule from the previous problem, and create multiple representations of each pair. Be sure to graph the function and its undo rule on the same set of axes.

When each person in your team has finished, put everyone's work into the middle of the workspace. Describe what relationships you see between the representations of a function and its undo rule.

November 28, 2010

6-6. What **strategies** did your team use to find undo rules? How can you be sure that the undo rules you found are correct? Discuss this idea and then write a Learning Log entry about the **strategies** you have for finding undo rules and checking that they work. Title this entry "Finding and Checking Undo Rules" and label it with today's date.



November 28, 2010

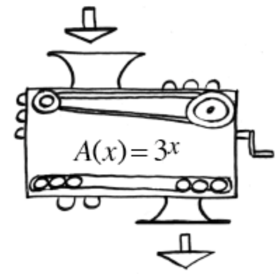
Review & Preview

6-7. Graph $y = \frac{1}{2}x - 3$ and its undoing function on the same set of axes.

a. What is the equation of the undoing function?

b. Does this graph, including both lines, have a line of symmetry? If so, what is the equation of the line of symmetry?

6-8. Antonio's function machine is shown at right.



a. What is $A(2)$?

b. If 81 came out, what was dropped in?

c. If 8 came out, what was dropped in? Be accurate to two decimal places.

6-9. Nossis has been working on his geometry homework and he is almost finished. His last task is to find a solution of $\sin(x) = 0.75$. Nossis cannot figure out what x could be! Explain how he can find a value for x and show that it works.

6-10. If $10^x = 10^y$, what is true about x and y? **Justify** your answer.

6-11. Solve each of the following equations for x.

a. $\frac{x}{3} = \frac{4}{5}$

b. $\frac{x}{x+1} = \frac{5}{7}$

c. $\frac{6}{15} = 2 - \frac{x}{5}$

d. $\frac{2}{3} + \frac{x}{5} = 6$

6-12. Sketch the solution of this system of inequalities.

$$y \geq x^2 - 5$$
$$y \leq -(x-1)^2 + 7$$

6-13. Gary has his function $g(x) = 10^x$ and Amy has her function $a(m) = 10^m$.

a. Each person is going to choose a whole number at random from the numbers 1, 2, 3…10, and substitute it into his or her respective function. After they do this, what is the probability that $g(x) = a(m)$?

b. Find and simplify an expression for $g(x) \cdot a(m)$.

6-14. Jamilla collected data comparing the weight and cost of pieces of sterling silver jewelry. Her data is listed as (weight in ounces, cost in dollars): (5, 44.00), (8.5, 78.50), (12, 112.00), (10, 93.00), (7, 63.50), (9, 83.20).

a. Plot the data on a set of axes.

b. Use a ruler to draw a line that best approximates the data.

c. Determine the equation of the line of best fit drawn in (b).

d. Use your equation to predict the cost of a 50-ounce silver bracelet.

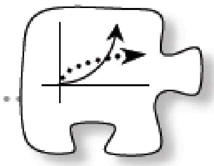
6-15. The angle of elevation of the sun (the angle the rays of sunlight make with the flat ground) at 10:00 a.m. is 29°. At that point, a tree's shadow is 32 feet long. How tall is the tree?

Lesson 6.1.1 Resource Page

I'm thinking of a number that…

when I…	*I get…*	
• triple my number and • add four	ten	
• double my number • add four and • divide by two	five	
• square my number • add three • divide by two and • add one	seven	
• double my number • subtract six • take the square root and • add four	eight	

6.1.2 How can I find the inverse?



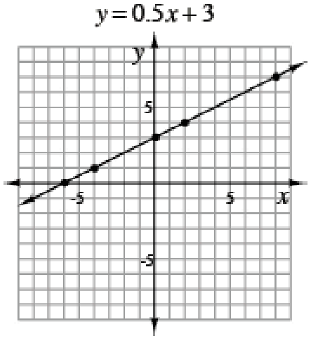
Using a Graph to Find an Inverse

What factors would you consider if you were thinking about buying a car? The first things that come to mind might be color or cost, but increasingly people are considering fuel efficiency (the number of miles a car can drive on a gallon of gas). You can think of the average number of miles per gallon that a car gets as a function that has *gallons* as the input and *miles traveled* as the output. A graph of this function would allow you to use what you know about the number of gallons in your tank to predict how far you could travel.

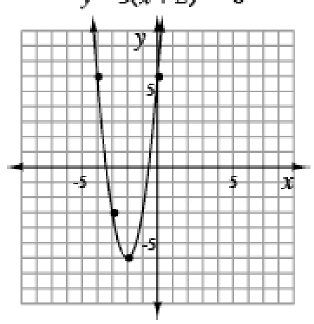
What would happen if you wanted to look at this situation differently? Imagine you regularly travel a route where there are many miles between gas stations. In this scenario, you would start with the information of the number of miles to the next filling station, and want to determine how many gallons of gas you would need to get there. In this case, you would start with the number of miles and work backwards to find gallons. Your new function would **reverse** the process.

6-16. In Lesson 6.1.1 you started with functions and worked backwards to find their undo rules. These undo rules are also called **inverses** of their related functions. Now you will focus on functions and their inverses represented as graphs. Use what you discovered yesterday as a basis for answering the questions below.

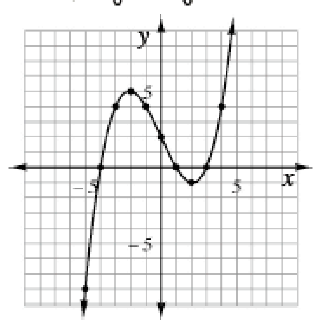
$y = 0.5x + 3$



$y = 3(x+2)^2 - 6$



$y = \frac{1}{6}x^3 - \frac{13}{6}x + 2$



a. Obtain a Lesson 6.1.2 Resource Page from your teacher and make a careful graph of each undo rule on the same set of axes as its corresponding function. Look for a way to make the graph without finding the undo rule first. Be prepared to share your **strategy** with the class.

b. Make statements about the relationship between the coordinates of a function and the coordinates of its inverse. Use $x \rightarrow y$ tables of the function and its inverse to show what you mean.

6-17. When you look at the graph of a function and its inverse, you can see a symmetrical relationship between the two graphs demonstrated by a line of symmetry.

a. Draw the line of symmetry for each pair of graphs in problem 6-16.

b. Find the equations of the lines of symmetry.

c. Why do you think these lines make sense as the lines of symmetry between the graphs of a function and its inverse relation?

6-18. The line of symmetry you identified in problem 6-17 can be used to help graph the inverse of a function without creating an $x \rightarrow y$ table.

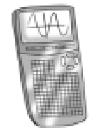
a. Graph $y = (\frac{x}{2})^2$ carefully on a full sheet of graph paper. Scale the x- and y-axes the same way on your graph.

b. On the same set of axes, graph the line of symmetry $y = x$.

c. With a pencil or crayon, trace over the curve $y = (\frac{x}{2})^2$ until the curve is heavy and dark. Then fold your paper along the line $y = x$, with the graphs on the inside of the fold. Rub the graph to make a "carbon copy" of the parabola.

d. When you open the paper you should see the graph of the inverse. Fill in any pieces of the new graph that did not copy completely. **Justify** that the graphs you see are inverses of each other.

6-19. Your graphing calculator can also help you to graph the inverse of a function. Check your inverse graph from problem 6-18 by following your teacher's instructions to use the "DrawInv" feature of your graphing calculator. Was the inverse graph that you drew correct?



6-20. Find the equation of the inverse of $y = (\frac{x}{2})^2$. Is there another way you could write it? If so, show how the two equations are the same. **Justify** that your inverse equation undoes the original function and use a graphing calculator to check the graphs.

6-21. Consider your equation for the inverse of $y = (\frac{x}{2})^2$.

a. Is the inverse a function? How can you tell?

b. Use color to trace over the portion of your graph of $y = (\frac{x}{2})^2$ for which $x \geq 0$. Then use another color to trace the inverse of *only this part* of $y = (\frac{x}{2})^2$. Is the inverse of this part of $y = (\frac{x}{2})^2$ a function?

c. Find a rule for the inverse of the restricted graph of $y = (\frac{x}{2})^2$. How is this rule different from the one you found in problem 6-20?

6-22. Consider the function $f(x) = (x-3)^2$.

a. How could you restrict the domain of $f(x)$ so that its inverse will be a function?

b. Graph $f(x)$ with its restricted domain and then graph its inverse on the same set of axes.

c. Find the equation of the inverse of $f(x)$ with its restricted domain.

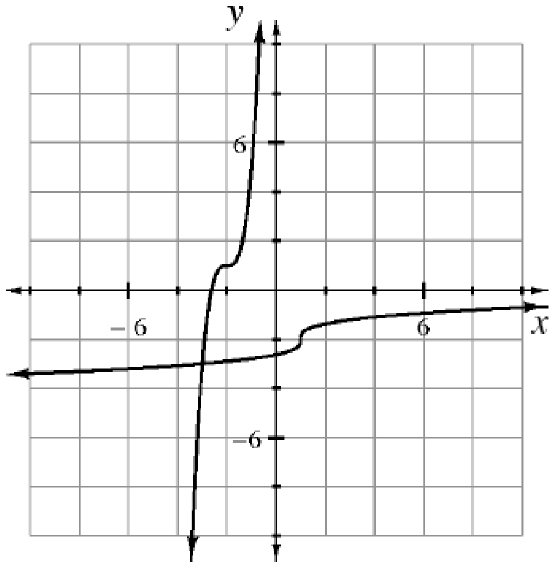
6-23. Is there a way to look at any graph to determine if its inverse will be a function? Explain. Find examples of other functions whose inverses are not functions.

6-24. Use graphs to find the inverses for the following functions. Label the graph of each function and its inverse with its equation.

a. $y = 5(x - 2)$

b. $y = 1 + \frac{2}{x}$

6-25. Look at the graph at right of a function and its inverse. If $p(x)$ is a function and $q(x)$ is its inverse, can you tell which is which? Why or why not?



MATH NOTES

METHODS AND MEANINGS

Notation for Inverses

When given a function $f(x)$, the notation for the inverse of the function is $f^{-1}(x)$. For example, if $f(x)=x^3-1$ then $f^{-1}(x)=\sqrt[3]{x+1}$.

Many calculators use this notation to identify the inverse of trigonometric functions. For example the inverse of $\sin(x)$ is written $\sin^{-1}(x)$.

Review & Preview

6-26. Make a graph of $f(x)=\frac{1}{2}(x-1)^3$ and then graph its inverse on the same set of axes.

6-27. Solve the system of equations at right.

$$\begin{aligned} x+y&=-3 \\ 2x-y&=-6 \\ 3x-2y+5z&=16 \end{aligned}$$

6-28. Solve the equation $3=8^x$ for x, accurate to the nearest hundredth (two decimal places).

6-29. Write the equation of a circle with a center at (–3, 5) that is tangent to the y-axis (in other words, it touches the y-axis at only one point). Sketching a picture will help.

6-30. Perform the indicated operation to simplify each of the following expressions. In some cases, factoring may help you simplify.

a. $\frac{(x+2)(x-3)}{(x+1)(x-4)}\cdot\frac{(x+1)}{x(x+2)}$

b. $\frac{x^2+5x+6}{x^2-4}\cdot\frac{4}{x+3}$

c. $\frac{2x}{x+4}+\frac{8}{x+4}$

d. $\frac{x}{x+1}-\frac{1}{x+1}$

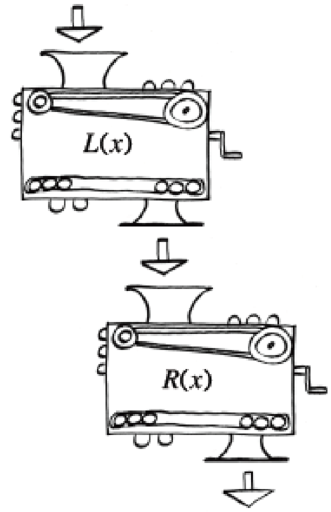
6-31. Barnaby's grandfather is always complaining that back when he was a teenager, he used to be able to buy his girlfriend dinner for only $1.50.

a. If that same dinner that Barnaby's grandfather purchased for $1.50 sixty years ago now costs $25.25, and the price has increased exponentially, write an equation that will give you the costs at different times.

b. How much would you expect the same dinner to cost in 60 years?

6-32. Ever eat a maggot? Guess again! The FDA publishes a list, the Food Defect Action Levels list, which indicates limits for "natural or unavoidable" substances in processed food (*Time*, October 1990). So in 100 g of mushrooms, for instance, the government allows 20 maggots! The average batch of rich and chunky spaghetti sauce has 350 grams of mushrooms. How many maggots does the government allow in a batch?

6-33. Lacey and Richens each have their own personal function machines. Lacey's, $L(x)$, squares the input and then subtracts one. Richen's function, $R(x)$, adds 2 to the input and then multiplies the result by three.



a. Write the equations that represent $L(x)$ and $R(x)$.

b. Lacey and Richens decide to connect their two machines, so that Lacey's output becomes Richens' input. Eventually, what is the output if 3 is the initial input?

c. What if the order of the machines was changed? Would it change the output? **Justify** your answer.

6-34. Solve the system of equations at right.

$$x-2y=7$$
$$6y-3x=33$$

a. What happened? What does this mean?

b. What does the solution tell you about the graphs?

6-35. Dana's mother gave her \$175 on her sixteenth birthday. *"But you must put it in the bank and leave it there until your eighteenth birthday,"* she told Dana. Dana already had \$237.54 in her account, which pays 3.25% annual interest, compounded quarterly. What is the minimum amount of money she will have on her eighteenth birthday if she makes *no* withdrawals before then? **Justify** your answer.

6-36. Consider the function $f(x)=\frac{2}{7-x}$.

a. What is $f(7)$?

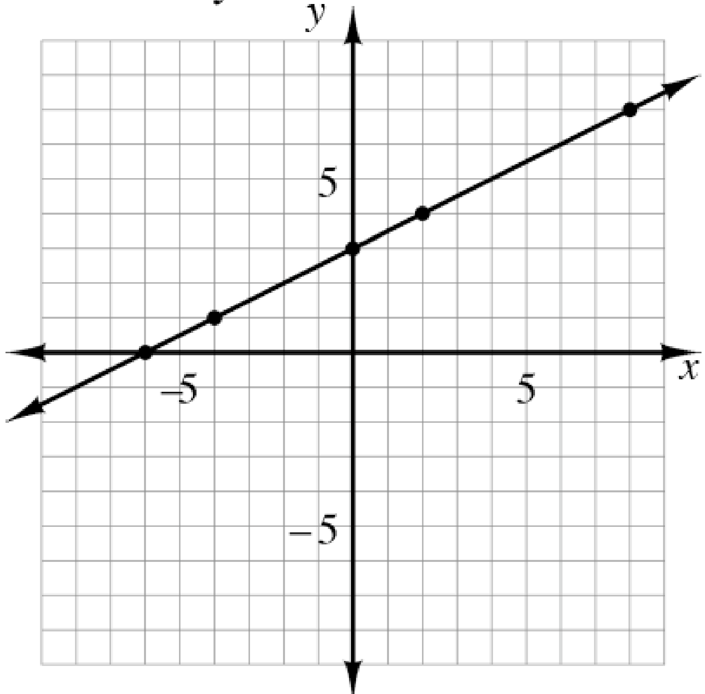
b. What is the domain of $f(x)$?

c. If $g(x)=2x+5$, what is $g(3)$?

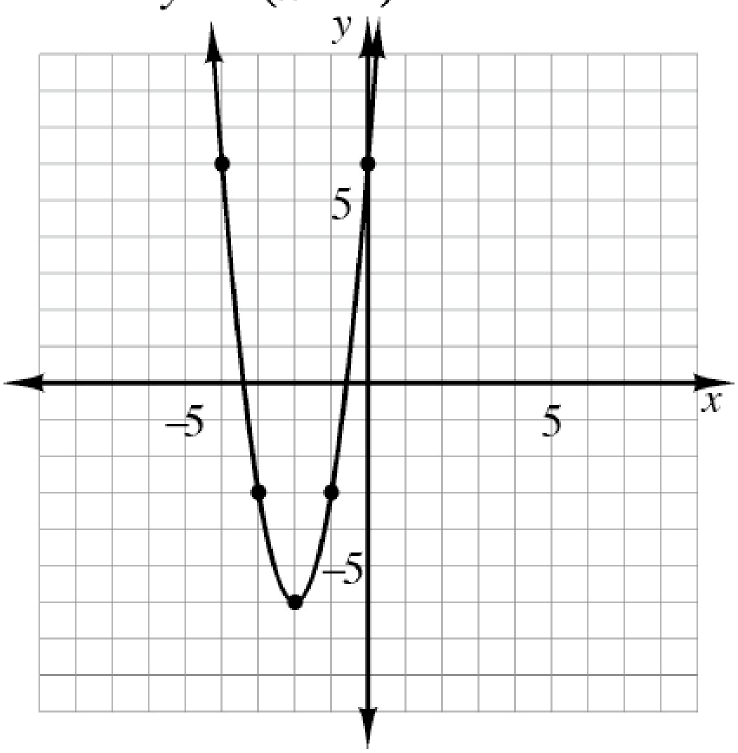
d. Now use the output of $g(3)$ as the input for f to calculate $f(g(3))$.

6-37. If $2^{x+4}=2^{3x-1}$, what is the value of x?

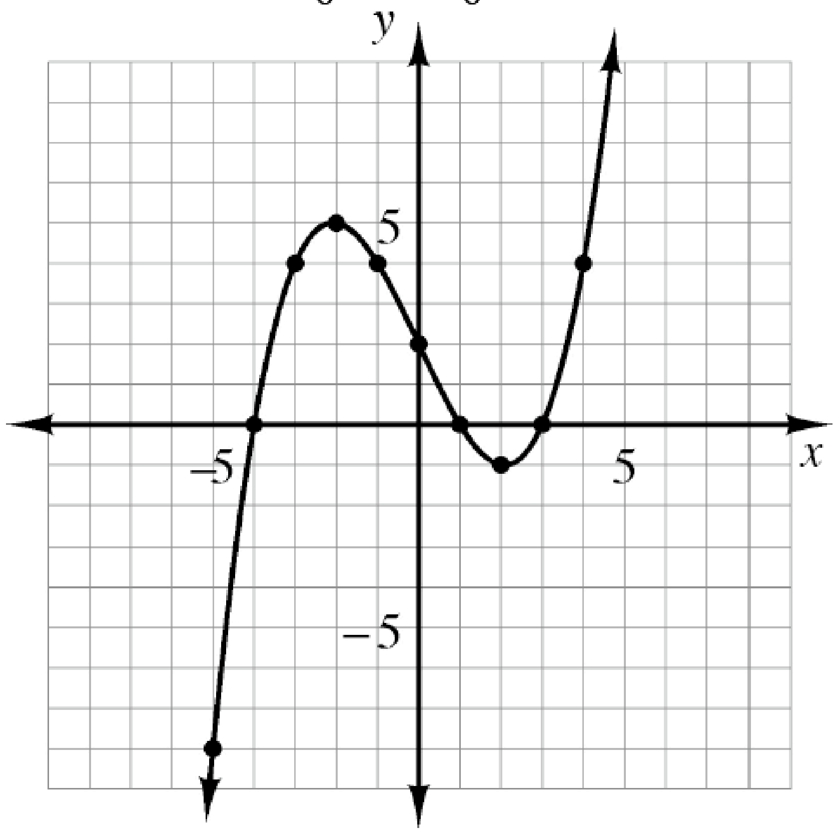
$y = 0.5x + 3$



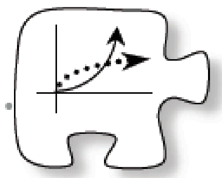
$y = 3(x+2)^2 - 6$



$$y = \frac{1}{6}x^3 - \frac{13}{6}x + 2$$



6.1.3 What can I do with inverses?



Finding Inverses and Justifying Algebraically

In this chapter you first learned how to find an inverse by undoing a function, and then you learned how to find an inverse graphically. You and your team may also have developed other **strategies**. In this lesson you will determine how to find an inverse by putting the ideas together and rewriting the equation. You will also learn a new way to combine functions that you can use to decide whether they have an inverse relationship.

6-38. Consider the table at right.

x	y
1	−5
3	7
5	19
7	31

a. Write an equation for the relationship represented in the table.

b. Make a table for the inverse.

c. How are these two tables related to each other?

d. Use the relationship between the tables to find a shortcut for changing the equation of the original function into its inverse.

e. Now solve this new equation for y.

f. **Justify** that the equations are inverses of each other.

6-39. Find the inverse function of the following functions using your new algebraic method, clearly showing all your steps.

a. $y = 2(x-1)^3$

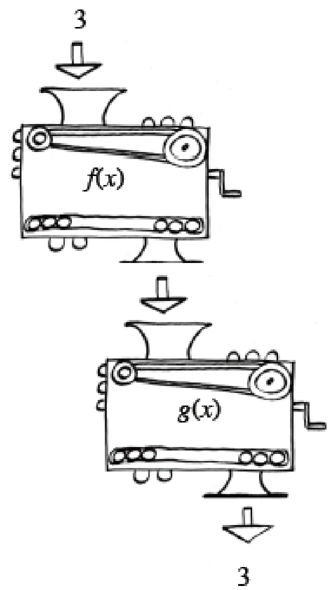
b. $y = \sqrt{x-2} + 3$

c. $y = 3(\frac{x-9}{2}) + 20$

d. $y = \frac{4}{3}(x-1)^3 + 6$

6-40. Adriena's **strategy** for checking that the functions $f(x)$ and $g(x)$ are inverses is to think of them as stacked function machines. She starts by choosing an input to drop into $f(x)$. Then she drops the output from $f(x)$ into $g(x)$. If she gets her original number, she is pretty sure that the two equations are inverses.

a. Is Adriena's **strategy** sufficient? Is there anything else she should test to be sure?



b. With your team, select a pair of inverse equations from problem 6-39, name them $f(x)$ and $g(x)$, then use Adriena's ideas to test them.

c. Adriena wants to find a shortcut to show her work. She knows that if she chooses her input for $f(x)$ to be 3, she can write the output as $f(3)$. Next, $f(3)$ becomes the input for $g(x)$, and her output is 3. Since $f(3)$ is the new input for $g(x)$, she thinks that she can write this process as $g(f(3)) = 3$. Does her idea make sense? Why or why not?

d. Her friend, Cemetra thinks she could also write $f(g(3))$. Is Cemetra correct? Why or why not.

e. Will this **strategy** for testing inverses work with any input? Choose a variable to use as an input to test with your team's functions, $f(x)$ and $g(x)$.

6-41. Christian, Adriena's teammate, is always looking for shortcuts. He thinks he has a way to adapt Adriena's **strategy**, but wants to check with his team before he tries it. *"If I use her strategy but instead of using a number, I skip a step and put the expression $f(x)$ directly into $g(x)$ to create $g(f(x))$, will I still be able to show that the equations are inverses?"*

a. What do you think about Christian's changes? What can you expect to get out?

b. Try Christian's idea on your team's equations, $f(x)$ and $g(x)$.

c. Describe your results.

d. Does Christian's **strategy** show that the two equations are inverses? How?

6-42. Adriena was finding inverses of some equations. Use Christian's **strategy** from Problem 6-41 to check Adriena's work and test if each pair of equations are inverses of each other. If they are not, explain what went wrong and show how to get the inverse correctly.

a. $f(x)=\frac{3}{5}x-15$
$g(x)=\frac{5}{3}x+25$

b. $f(x)=\frac{2(x+6)}{3}+10$
$g(x)=\frac{3}{2}x-21$

c. $e(x)=\frac{(x-10)^2}{4}$
$d(x)=4\sqrt{x}+10$

6-43. Make a personal poster that shows what you have learned about inverses so far. Choose an equation and its inverse then **justify** that your equations are inverses of each other using several representations.

MATH NOTES

METHODS AND MEANINGS

Composition of Functions

When we stack one function machine on top of another so that the output of the first machine becomes the input of the second, we create a new function, which is a **composition** of the two functions. If the first function is $g(x)$ and the second is $f(x)$, the composition of f and g can be written $f(g(x))$. (Note that the notations $f \circ g$ or $f \circ g(x)$ are used in some texts to denote the same composition.)

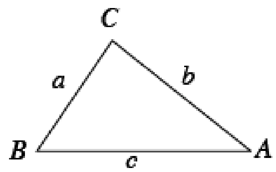
Note that the order of the composition matters. In general, the compositions $g(f(x))$ and $f(g(x))$ will be different functions.

MATH NOTES

METHODS AND MEANINGS

Laws of Sines and Cosines

For any uniquely determined triangle, missing sides and angles can be determined by using the **Law of Sines** or the **Law of Cosines**.



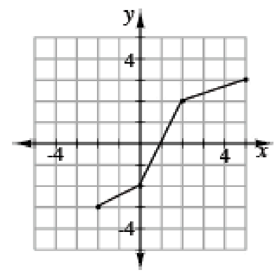
Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

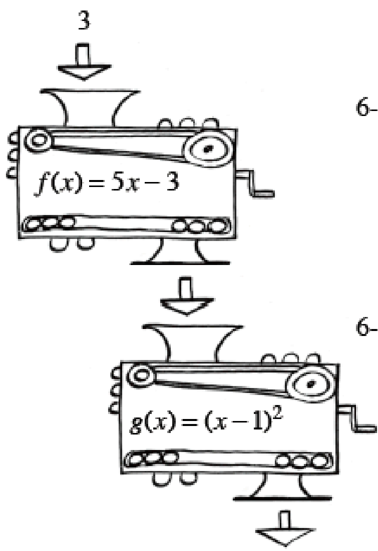
Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos C$

Review & Preview

6-44. Trejo says that if you know the x-intercepts, y-intercepts, domain, and range of an equation then you automatically know the x-intercepts, y-intercepts, domain, and range for the inverse. Hilary disagrees. She says you know the intercepts but that is all you know for sure. Who is correct? **Justify** your answer.

6-45. The function $f(x)$ is represented in the graph at right. Draw a graph of its inverse function. Be sure to state the domain and range for both $f(x)$ and $f^{-1}(x)$.





6-46. Two function machines, $f(x) = 5x - 3$ and $g(x) = (x-1)^2$, are shown at left. Suppose $f(3)$, (*not* $x = 3$), is dropped into the $g(x)$ machine. This is written as $g(f(3))$. What is this output?

6-47. Using the same function machines as in the previous problem, what is $f(g(3))$? Be careful! The result is different from the last one because the *order* in which you use the machines has been switched! With $f(g(3))$, first you find $g(3)$, then you substitute that answer into the machine named f.

6-48. This is a Checkpoint for working with integral and rational exponents.

Use integer or rational exponents to write each of the following expressions as an exponential expression with a base of x.

a. $\sqrt[5]{x}$ b. $\frac{1}{x^3}$ c. $\sqrt[3]{x^2}$ d. $\frac{1}{\sqrt{x}}$

e. Check your answers by referring to the Checkpoint 11 materials located at the back of your book.

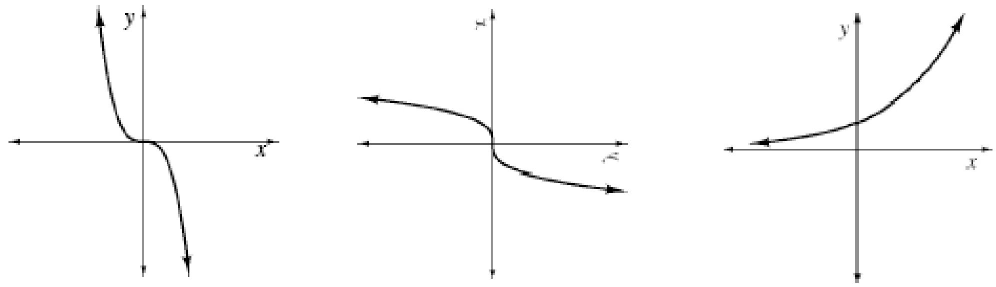
If you needed help to rewrite these expressions correctly, then you need more practice in simplifying expressions with integral or rational exponents. Review Checkpoint 11 materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to simplify expressions such as these easily and accurately.

6-49. Solve each of the following equations.

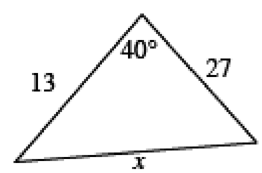
a. $\frac{3x}{5} = \frac{x-2}{4}$ b. $\frac{4x-1}{x} = 3x$

c. $\frac{2x}{5} - \frac{1}{3} = \frac{137}{3}$ d. $\frac{4x-1}{x+1} = x - 1$

6-50. Rebecca thinks that she has found a quick way to graph an inverse of a function. She figures that if you can interchange x and y to find the inverse, she will interchange the x- and y-axes by flipping the paper over so that when she looks through the back the x-axis is vertical and the y-axis is horizontal as shown in the pair of graphs below left. Copy the graph on the right onto your paper and try her technique. Does it work? If so, do you like this method? Why or why not?



6-51. Find the value of x. Refer to the Math Notes box above, for a reminder of the Laws of Sines and Cosines. **Justify** how you know your answer is reasonable.



6-52. Complete the square to write $x^2 + y^2 - 4x - 16 = 0$ in graphing form and sketch the graph.

6-53. Perform each operation below and simplify your results.

a. $\dfrac{x^2+4x+3}{x^2+3x} \cdot \dfrac{3x}{x+1}$

b. $\dfrac{y^2}{y+4} - \dfrac{16}{y+4}$

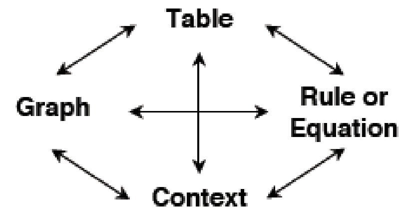
c. $\dfrac{x^2+x}{x^2-4x-5} \div \dfrac{3x^2}{x-5}$

d. $\dfrac{x^2-6x}{x^2-4x+4} + \dfrac{4x}{x^2-4x+4}$

6.2.1 How can I undo an exponential function?

Finding the Inverse of an Exponential Function

When you first began **investigating exponential** functions you looked at how their different representations were interconnected, as in the web at right. So far in this chapter you have considered how functions and their inverses are related in different representations including rules, $x \to y$ tables, and graphs. What would the inverse relation for each of the parent functions you worked with in Chapter 4 look like in each representation?



As you work with your team today, ask each other these questions:

What does the parent function look like in this representation?
How can that help us see the inverse relation?

Would another representation be more helpful?

How can we describe the relationship in words?

6-54. So far, you have learned a lot about eight different parent graphs:

i. $y = x^2$ *ii.* $y = x^3$ *iii.* $y = x$ *iv.* $y = |x|$

v. $y = \sqrt{x}$ *vi.* $y = \frac{1}{x}$ *vii.* $y = b^x$ *viii.* $x^2 + y^2 = 1$

a. For each parent, find the inverse. Be sure to write the equation of the inverse in y-form, if possible. Include a sketch of each parent graph and its inverse. Remember that you can use DrawInv on your graphing calculator to help test your ideas.

b. Are any parent functions their own inverses? Explain how you know.

c. Do any parent functions have inverses that are not functions? If so, which ones?

6-55. THE INVERSE EXPONENTIAL FUNCTION

There are two parent functions, $y=|x|$ and $y=b^x$, that have inverses that you do not yet know how to write in y-form. You will come back to $y=|x|$ later. Since exponential functions are so useful for modeling situations in the world, the inverse of an exponential function is also important. Use $y=3^x$ as an example. Even though you may not know how to write the inverse of $y=3^x$ in y-form, you know a lot about it.

a. You know how to make an $x \to y$ table for the inverse of $y=3^x$. Make the table.

b. You also know what the graph of the inverse looks like. Sketch the graph.

c. You also have one way to write the equation based on your algebraic shortcut from problem 6-38 (d). Write an equation for the inverse, even though it may not be in y-form.

d. If the input for the inverse function is 81, what is the output? If you could write an equation for this function in y-form, or as a function $g(x)=$, and you put in any number for x, how would you describe the outcome?

6-56. AN ANCIENT PUZZLE

Parts (a) – (f) below are similar to a puzzle that is more than 2100 years old. Mathematicians first created the puzzle in ancient India in the 2nd century BC. More recently, about 700 years ago, Muslim mathematicians created the first tables allowing them to find answers to this type of puzzle quickly. Tables similar to them appeared in school math books until recently.

Here are some clues to help you figure out how the puzzle works:

$\log_2 8 = 3$ $\qquad$ $\log_3 27 = 3$

$\log_5 25 = 2$ $\qquad$ $\log_{10} 10{,}000 = 4$

Use the clues to find the missing pieces of the puzzles below:

a. $\log_2 8 = ?$

b. $\log_2 32 = ?$

c. $\log_? 100 = 2$

d. $\log_5 ? = 3$

e. $\log_? 81 = 4$

f. $\log_{100} 10 = ?$

6-57. How is the Ancient Puzzle related to the problem of the inverse function for $y = 3^x$ in problem 6-55? Show how you can use the idea in the Ancient Puzzle to write an equation in y-form or as $g(x) =$ for the inverse function in problem 6-55.

6-58. THE INVERSE OF ABSOLUTE VALUE

a. Find the inverse equation and graph of $y = 2|x+1|$.

b. Although you know how to find the table, graph, and equation for the inverse of absolute value, this is another function whose inverse equation cannot easily be written in y-form. In fact, there is no standard notation for the inverse of the absolute value function. With your team, invent a symbol to represent the inverse, and give examples to show how your symbol works. Be sure to explain how your symbol handles that fact that the inverse of $y = |x|$ is not a function or explain why it is difficult to come up with a reasonable notation.

Review & Preview

6-59. In problem 6-55, you looked at the inverse of $y = 3^x$. Finish **investigating** this function.

6-60. Amanda wants to showcase her favorite function: $f(x) = 1 + \sqrt{x+5}$. She has built a function machine that performs these operations on the input values. Her brother Eric is always trying to mess up Amanda's stuff, so he created the inverse of $f(x)$, called it $e(x)$, and programmed it into a machine.

a. What is Eric's equation for his function $e(x)$?

b. What happens if the two machines are pushed together? What is $e(f(-4))$? Explain why this happens.

c. If $f(x)$ and $e(x)$ are graphed on the same set of axes, what would be true about the two graphs?

d. Draw the two graphs on the same set of axes. Be sure to show clearly the restricted domain and range of Amanda's function.

6-61. Sketch the graph of $y + 3 = 2^x$.

a. What are the domain and range of this function?

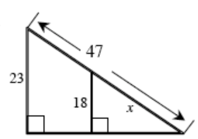
b. Does this function have a line of symmetry? If so, what is it?

c. What are the x- and y-intercepts?

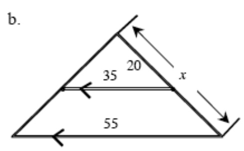
d. Change the equation so that the graph of the new equation has no x-intercepts.

6-62. Solve for x in the following diagrams.

a.



b.



6-63. Sketch square $ABCD$ on your paper, then randomly choose a point on $\overline{AB}$ and label it X. Draw $\overline{XC}$ and $\overline{XD}$ to form ΔXCD. If a dart is thrown and lands inside the square, what is the probability that it landed inside ΔXCD? Does it matter where you place X on $\overline{AB}$?

6-64. A woman plans to invest x dollars. Her investment counselor advises her that a safe plan is to invest 30% of that money in bonds and 70% in low risk stocks. The bonds currently have a simple interest rate of 7% and the stock has a dividend rate (like simple interest) of 9%.

a. Write an expression for the annual income that will come from the bond investment.

b. Write an expression for the annual income that will come from the stock investment.

c. Write an equation and solve it to find out how much the client needs to invest to have an annual income of $5,000.

6-65. Some of the following algebraic fractions have common denominators and some do not. Add or subtract the expressions and simplify, if possible.

a. $\frac{3}{(x-4)(x+1)} + \frac{6}{x+1}$

b. $\frac{5}{2(x-5)} + \frac{3x}{x-5}$

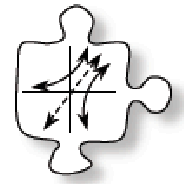
c. $\frac{x}{x^2-x-2} - \frac{2}{x^2-x-2}$

d. $\frac{x+2}{x^2-9} - \frac{1}{x+3}$

6-66. Sketch the solution to this system of inequalities.

$$y \geq (x+5)^2 - 6$$
$$y \leq -(x+4)^2 - 1$$

6.2.2 What is a logarithm?



Defining the Inverse of an Exponential Function

So far, you have learned how to "undo" many different functions. However, the exponential function has posed more difficulty. In this lesson, you will learn more about the inverse exponential function. In particular, you will learn how to write an inverse exponential function in y-form.

6-67. SILENT BOARD GAME

Your teacher will put an $x \to y$ table on the board or overhead that the whole class will work together to complete. The table will be like the one below. See which values you can fill in.

x	8	32	$\frac{1}{2}$	1	16	4	3	64	2	0	0.25	-1	$\sqrt{2}$	0.2	$\frac{1}{8}$
$g(x)$	3		-1					6					$\frac{1}{2}$		

a. Describe a rule that relates x and $g(x)$.

b. Look back to the Ancient Puzzle in problem 6-55. If you haven't already, use the idea of the Ancient Puzzle to write an equation for $g(x)$.

c. Why was it difficult to think of an output for the input of 0 or -1?

d. Find an output for $x = 25$ to the nearest hundredth.

6-68. ANOTHER LOGARITHM TABLE

Lynn was supposed to fill in this table for $g(x) = \log_5 x$. She thought she could use the log button on her calculator, but when she tried to enter 5, 25, and 125, she did not get the ouputs the table below displays. She was fuming over how long it was going to take to guess and check each one when her sister suggested that she did not have to do that for all of them. She could fill in a few more and then use what she knew about exponents to figure out some of the others.

x	$\frac{1}{25}$	$\frac{1}{5}$	$\frac{1}{2}$	1	2	3	4	5	6	7	8	10	25	100	125	625
$g(x)$		–1		0				1					2		3	

a. Discuss with your team which outputs can be filled in without a calculator. Fill those in and explain how you found these entries.

b. With your team, use your calculator to estimate the remaining values of $g(x)$ to the nearest hundredth. Once you have entered several, use your knowledge of exponent rules to see if you can find any shortcuts.

c. What do you notice about the results for $g(x)$ as x increases?

d. Use your table to draw the graph of $y = \log_5 x$. How does your graph compare to the graph of $y = 5^x$?

6-69. Find each of the values below, and then **justify** your answers by writing the equivalent exponential form.

a. $\log_2(32) = ?$

b. $\log_2(\frac{1}{2}) = ?$

c. $\log_2(4) = ?$

d. $\log_2(0) = ?$

e. $\log_2(?) = 3$

f. $\log_2(?) = \frac{1}{2}$

g. $\log_2(\frac{1}{16}) = ?$

h. $\log_2(?) = 0$

6-70. While the idea behind the Ancient Puzzle is more than 2100 years old, the symbol **log** is more recent. It was created by John Napier, a Scottish mathematician in the 1600's. "Log" is short for **logarithm**, and represents the function that is the **inverse of an exponential function**. You can use this idea to find the inverse equations of each of the following functions. Find the inverses and write your answers in *y*-form.

a. $y = \log_9(x)$

b. $y = 10^x$

c. $y = \log_6(x+1)$

d. $y = 5^{2x}$

6-71. Practice your logarithm fluency by calculating each of the following, *without changing the expressions to exponential form*. Be ready to explain your thinking.

a. $\log_7 49 =$ __

b. $\log_3 81 =$ __

c. $\log_5 5^7 =$ __

d. $\log_{10} 10^{1.2} =$ __

e. $\log_2 2^{w+3} =$ __

MATH NOTES

METHODS AND MEANINGS

Logarithms and Their Notation

A **logarithm** (called a "log" for short) is an exponent. An expression in logarithmic form, such as $\log_2(32)$, is read, *"the log, base 2, of 32."* To evaluate log expressions, think of the exponent: $\log_2(32) = 5$, because the exponent needed for base 2 to become 32 is 5.

An equation in logarithmic form is equivalent to another equation in exponential form, as shown at right. This conversion helps show why (based on an $x \rightarrow y$ interchange) $y = \log_b(x)$ and $y = b^x$ are inverse functions.

$$y = \log_b(x)$$

$$b^y = x$$

Review & Preview

6-72. Let $y = \log_2(x)$. Rewrite the equation so that it begins with $x =$. Think about how you defined $y = \log_2(x)$ if you get stuck. Put a large box around both equations. Do the two equations look the same? Do the two equations mean the same thing? Are they equivalent? How do you know? This is very important. Think about it, and write a clear explanation.

6-73. Every exponential equation has an equivalent logarithmic form and every logarithmic equation has an equivalent exponential form. For example,

$$\underset{\text{base}}{4}^{\overset{\text{exponent}}{3}} = 64 \quad \text{is equivalent to} \quad \underset{\text{exponent}}{3} = \log_{\underset{\text{base}}{4}} 64$$

Copy the table shown below and fill in the missing form in each row.

	Exponential Form	Logarithmic Form
a.	$y = 5^x$	
b.		$y = \log_7(x)$
c.	$8^x = y$	
d.	$A^K = C$	
e.		$K = \log_A(C)$
f.		$\log_{1/2}(K) = N$

6-74. If $x = 7^y$, how would you write this equation in y-form? Explain.

6-75. Find the value of x in the equation $2^x = 3$. Be accurate to three decimal places.

6-76. Although the quadratic formula always works as a **strategy** to solve quadratic equations, for many problems it is not the most efficient method. Sometimes it is faster to factor or complete the square or even just "out-think" the problem. For each equation below, choose the method you think is most efficient to solve the equation and explain your reason. **Note that you do *not* actually need to solve the equation.**

a. $x^2 + 7x - 8 = 0$

b. $(x + 2)^2 = 49$

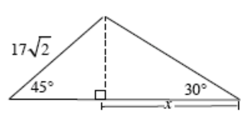
c. $5x^2 - x - 7 = 0$

d. $x^2 + 4x = -1$

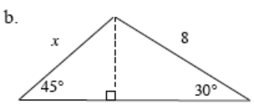
6-77. If $10^{3x} = 10^{(x-8)}$, solve for x. Show that your solution works by checking your answer.

6-78. Find the value of x in each diagram below.

a.



b.



6-79. Consider the function defined by inputs that are the length of the radii of a circle, and the outputs are the areas of those circles. Write the rule for this function and **investigate** it completely.

6-80. Consider the equation $y = (x + 6)^2 - 7$.

a. Explain completely how to get a good sketch of the graph of $y = (x + 6)^2 - 7$.

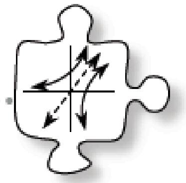
b. Explain how to change the original graph to represent the graph of $y = (x + 6)^2 + 2$.

c. Given the original graph, how can you get the graph of $y = \left|(x + 6)^2 - 7\right|$?

d. Restrict the domain of the original parabola to $x \geq -6$ and graph its inverse function.

e. What would be the equation for the inverse function if you restricted the domain to $x \geq -6$?

6.2.3 What can I learn about logs?



Investigating the Family of Logarithmic Functions

In the last two lessons you have learned what a log is and how to convert an equation in log form to exponential form (and back again). In this lesson, you will explore logs as a family of functions.

6-81. INVESTIGATING THE FAMILY OF LOGARITHMIC FUNCTIONS

You have learned that a logarithm is the inverse of an exponential function. Since exponential functions can have different bases, so can logarithms. **Investigate** the family of logarithmic functions $y = \log_b(x)$. The questions below will help you **investigate**.

Your task: Generate data with your team and use it to write summary statements about this family of functions. For each summary statement you find, prepare a transparency that shows and explains the summary statement and be prepared to present it to the class. Remember that summary statements should always include thorough **justification**.

Discussion Points

How can we collect data for this family? How much data is enough?

What have we learned about logs and inverses that can help us work with this family?
How can "DrawInv" help?

What patterns can we find in our data? Why do they happen?

What are all the possible inputs for our function? Are there some x-values
that do not make sense? Why or why not?
How do these results appear in different mathematical representations?

What are some characteristics that all logarithmic functions have in common?

What happens as the value of b changes? What values of b make sense?

Further Guidance

6-82. As a team, begin your **investigation** of $y = \log_b x$ by choosing a positive value for b and work together to generate a table and a graph. Then, have each member of your team choose a different value for b. Since there is no key for a log of base b on your calculator, you will need to find another method to generate data for a table. Several **strategies** are suggested below.

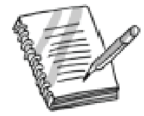
- While it may still be hard to make a table for your equation, your knowledge of inverses will help you. Write the inverse of your equation and make an $x \rightarrow y$ table for it. Use this table to help you make a table for your original function.

- Use the calculator to guess and check possible outcomes.

- Rewrite your log equation as an equivalent exponential equation and **reverse your thinking**.

Further Guidance section ends here.

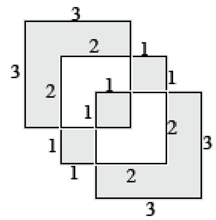
6-83. Write a Learning Log entry about the family of functions $y = \log_b x$. Include the summary statements your team came up with and any others that you think should be added from the class discuission. As you write, think about which statements are very clear to you and which need further clarification. Title this entry "The Family of Logarithmic Functions" and label it with today's date.



Review & Preview

6-84. Write the equation of an increasing exponential function that has a horizontal asymptote at $y = 15$.

6-85. If a point inside the figure at right is chosen at random, what is the probability that it is in the shaded region?



6-86. Solve for n: $n^3 = 49$.

6-87. A circle has the equation $x^2 + (y+2)^2 = r^2$. If the circle is shifted 2 units to the left, 5 units up, and the radius is doubled, what will its new equation be?

6-88. On Wednesdays at Tara's Taqueria four tacos are the same price as three burritos. Last Wednesday the Lunch Bunch ordered five tacos and six burritos, and their total bill was \$8.58 (with no tax or drinks included). Nobody in the Lunch Bunch can remember the cost of one of Tara's tacos. Help them figure it out.

6-89. Graph the two functions at right on the same set of axes.

$$y = 3(2^x)$$
$$y = 3(2^x) + 10$$

a. How do the two graphs compare?

b. Suppose the first equation is $y = km^x$ and the graph is shifted up b units. What is the new equation?

6-90. Solve each equation or inequality.

a. $|x-1| = 9$ 　　b. $2|x+1| + 3 = 9$

c. $|x-1| < 3$ 　　d. $|x+5| \ge 8$

6-91. For each of the following rational expressions, add or subtract, then simplify.

a. $\frac{2-x}{x+4} + \frac{3x+6}{x+4}$ 　　b. $\frac{3}{(x+2)(x+3)} + \frac{x}{(x+2)(x+3)}$

c. $\frac{3}{x-1} - \frac{2}{x-2}$ 　　d. $\frac{8}{x} - \frac{4}{x+2}$

6-92. Each step of a simplification process must be **justifiable** using the properties of algebra.

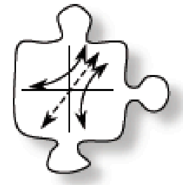
a. Examine the **justification** for each step in the simplification below.

Given expression: $2(x + \frac{3}{x}) - \frac{4}{x}$

Step 1:	$2x + \frac{6}{x} - \frac{4}{x}$	Distributive Property
Step 2:	$\frac{2x^2}{x} + \frac{6}{x} - \frac{4}{x}$	Multiplicative Identity ($1 \cdot a = a$)
Step 3:	$\frac{1}{x}(2x^2 + 6 - 4)$	Distributive Property
Step 4:	$\frac{2x^2+6-4}{x}$	Definition of Division ($a \div b = a(\frac{1}{b})$)
Step 5:	$\frac{2x^2+2}{x}$	Associative Property of Addition

b. Use the properties of algebra to **justify** each step in simplifying the expression in part (d) of problem 6-91.

6.2.4 How can I transform log functions?



Transformations of Logarithmic Functions

In Lesson 6.2.3, you **investigated** logarithmic functions with different bases. To do this, you had to convert a log equation into its corresponding exponential form. In this lesson, you will figure out what a graphing calculator can and cannot do with logs. This will help you write a general equation for a log function. As you work with your team, use the following questions to help focus your discussions.

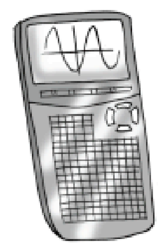
What is a log?

How are logarithms and exponential equations related to each other?

How can we find an equivalent exponential equation for an equation that is in log form?

How can we transform the graphs of log functions?

6-93. SOLVE THE LOG MYSTERY!



Have you noticed the $\boxed{\text{LOG}}$ key on your calculator? Clearly it is a logarithm, but what is its base? It would have been nice if the designers of your graphing calculator had allowed the $\boxed{\text{LOG}}$ key to work with any base, but they did not!

Your task: Find the base of the $\boxed{\text{LOG}}$ key on your calculator. With your team, start by gathering some data and making a table for $y = \log x$. Analyze your data, and when you are sure you have figured out the base, write a clear summary statement **justifying** your conclusion.

Discussion Points

What input values give whole number outputs?
What do those values tell us?

How can we rewrite $y = \log_? x$?

6-94. Now that you know the base of $f(x) = \log x$, you are ready to use your transformation skills to write a general equation.

a. Copy and complete the following table for $f(x) = \log x$.

x								1	2	3	4	5	6
y	–6	–5	–4	–3	–2	–1	0						

b. Using a full sheet of graph paper, make an accurate graph of $f(x) = \log(x)$. Remember that, just like the graphs of exponential functions, the graphs of log functions have asymptotes, so make sure any asymptotes on your graph are clearly shown.

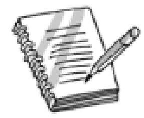
c. Find all of the possible types of transformations of the graph of $f(x) = \log x$. For each transformation you find, show the graph and its equation. Then, find the general form for this family of logarithm graphs. Be prepared to explain your reasoning to the class.

6-95. You have learned a lot about logs in a short time. Use what you have learned so far to answer the questions below.

a. Why does your calculator say that $\log(6) \approx 0.778$?

b. **Justify** why $\log(6)$ must have a value less than 1 but greater than 0.

c. Create a Learning Log entry that includes your answers to the focus questions from today's lesson, reprinted below. Show examples and use color or arrows to help explain your ideas. Title this entry "Working with Logs" and label it with today's date.



What is a log?

How are logarithms and exponential equations related to each other?

How can you find an equivalent exponential equation for an equation that is in log form?

How can you transform log functions?

Review & Preview

6-96. Last night, while on patrol, Agent 008 came upon a spaceship! He hid behind a tree and watched a group of little space creatures carry all sorts of equipment out of the ship. But suddenly, he sneezed. The creatures jumped back into their ship and sped off into the night. 008 noticed that they had dropped something so he went to pick it up. It was a calculator! What a great find. He noticed that it had a LOG button, but he noticed something interesting: log 10 did not equal 1! With this calculator, $\log 10 \approx 0.926628408$. He tried some more: $\log 100 \approx 1.853256816$ and $\log 1000 \approx 2.779885224$.

a. What base do the space creatures work in? Explain how you can tell.

b. How many fingers do you think the space creatures have?

6-97. Copy these equations and solve for x. You should be able to do all these problems without a calculator.

a. $\log_x(25) = 1$ b. $x = \log_3(9)$

c. $3 = \log_7(x)$ d. $\log_3(x) = \frac{1}{2}$

e. $3 = \log_x(27)$ f. $\log_{10}(10000) = x$

6-98. Is $\log(0.3)$ greater than or less than one? **Justify** your anwer.

6-99. Solve $1.04^x = 2$. Your answer should be accurate to three decimal places.

6-100. Perform each operation below and simplify your results.

a. $\frac{x^2+5x+6}{x^2-4x} \cdot \frac{4x}{x+2}$ b. $\frac{x^2-2x}{x^2-4x+4} \div \frac{4x^2}{x-2}$

6-101. Solve the following inequalities.

a. $x^2 - 2x < 3$ b. $3x - x^2 \le 2$

6-102. Solve for m: $m^5 = 50$.

6-103. Is it true that $\log_3(2) = \log_2(3)$? **Justify** your answer.

6-104. Consider the general form of an exponential function: $y = ab^x$.

a. Solve for a. b. Solve for b.

6-105. Make a sketch of a graph that is a decreasing exponential function with the x-axis as the horizontal asymptote. Then make a similar sketch, but this time with the line $y = 5$ as the horizontal asymptote.

6.2.5 How can I build a new function?

Investigating Compositions of Functions

Today you will work with your team to create and analyze new, interesting functions that are compositions of functions with which you are already familiar.

6-106. Polly Parabola's first corporate venture, Professional Parabola Productions, was so successful that Felix's Famous Functions bought her out in a corporate takeover. With all of the money she made from the transaction, she has decided to start a new company, Creative Compositions. Creative Compositions plans to develop a line of composite functions designed to appeal to the imagination of the next generation of function groupies. She wants to market three new functions and is offering huge contracts to the winners of the competition. Your boss wants your company to enter this competition and has assigned your team to do the development.

CREATIVE COMPOSITIONS

Call for new and visually interesting
Compositions of functions

The Creative Composition Corporation announces an open competition for contracts to design new products. The products must be a composition of two or more functions whose parent functions are listed below:

$f(x)=x^2$ $g(x)=x^3$ $h(x)=b^x$ $i(x)=\frac{1}{x}$ $j(x)=\sqrt{x}$ $k(x)=|x|$ $l(x)=\log_b x$

Competing teams will prepare a poster to display their composite function and respond to questions from a panel of judges. Three contracts will be awarded based on the evaluation of the judges.

The judges will base their review on the following:

Is the graph of the composition a new and interesting shape?

Are multiple representations used effectively
to show key features of the new function?

Does the selection of examples show off a variety of ways
the function will appear when it is transformed?

6-106. *Problem continued from previous page.*

Your task: With your team, try out different ways to write compositions involving two or more of the given functions and check their graphs. Record everything you try as documentation for the report you will need to give your boss. When your team agrees on a function they like, **investigate** it thoroughly and prepare a poster for the competition.

Discussion Points

What does the graph of each function look like separately?

How does making the output of one function the input of the other change the original graph?

How do we have to adjust the domains and ranges?

Is the inverse a function?

Further Guidance

6-107. Consider $f(x)=2^x$ and $k(x)=|x|$. Write the rule for each composite function $k(f(x))$ and $f(k(x))$. Discuss what each graph will look like and then sketch it. For each graph, explain the effect of one parent function on the other.

6-108. Choose other pairs of parent functions from the list. Then write the composite functions in both directions. In other words, use one function as the input for the other and then switch. Check the graphs and decide whether either is a good candidate for the competition. Try out at least five different pairs and record your equations and sketches of their graphs.

6-109. As a team, decide which of the functions you created that you want to enter in the competition. Now do a thorough **investigation** of that function.

6-110. Prepare a poster to show off your new function. Be sure to include all of the important details from your **investigation** on your poster and be prepared to respond to the judges with your arguments for why this function should be selected as one of the new products of Creative Compositions.

Further Guidance section ends here.

6-111. Consider the functions $f(x)$ in parts (a) and (b) below. For each $f(x)$, find two functions $h(x)$ and $g(x)$, so that $h(g(x)) = f(x)$. Use numerical examples to demonstrate that your functions $h(x)$ and $g(x)$ work.

a. $f(x) = \sqrt{3x+6}$

b. $f(x) = \frac{5}{\sqrt{x}}$

c. **Challenge:** Work with your team to find another possibility for $h(x)$ and $g(x)$ such that $h(g(x)) = f(x)$ for each function given in parts (a) and (b). Be prepared to share your ideas with the class.

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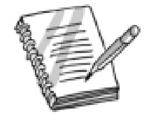
6-112. Create a Learning Log entry explaining what you have learned about compositions of functions. Use examples to illustrate your ideas. Title this entry "Compositions of Functions" and label it with today's date.



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Review & Preview

6-113. If $f(x)=\sqrt{7-x^2}-6$ and $g(x)=-(x+6)^2+7$, find $f(g(x))$ and $g(f(x))$. What do the results tell you about $f(x)$ and $g(x)$?

6-114. For functions of the form $f(x)=mx$, it is true that $f(a)+f(b)=f(a+b)$? For example, when $f(x)=5x$, $f(a)+f(b)=5a+5b=5(a+b)$ and $f(a+b)=5(a+b)$. Is $f(a)+f(b)=f(a+b)$ true for all linear functions? Explain why or show why not.

6-115. Consider the following three sequences:

$t(n)=50-7n$ $\qquad$ $h(n)=4\cdot 3^n$ $\qquad$ $q(n)=n^2-6n+17$

a. Which, if any, is arithmetic? Geometric? Neither?

b. Are there any terms that all three sequences have in common? **Justify** how you know for sure.

c. Are there any terms that two of them share? **Justify** how you know for sure.

6-116. Using the sequences in the previous problem, suppose we define a new sequence, $s(n)$, defined as $s(n)=q(t(n))$, a compostion of two sequences. Do you think the new sequence will be arithmetic? Geometric? Neither? Explain. Make a table of values. Does the table support your hypothesis, or do you want to change your guess? Explain.

6-117. Sketch the graph of $y=3\log(x+4)-1$.

6-118. Consider two functions $f(x)=\log x$ and $g(x)=|x|$.

a. Use these two functions to write an equation for a composite function and sketch its graph.

b. Use these two functions to write a different composite function and sketch its graph.

c. What makes the two composite functions so different from each other?

d. **Challenge:** Now try graphing $g(f(g(x)))$.

6-119. Solve $5^x=15$ for x. Be accurate to two decimal places.

6-120. Simplify each of the expressions in parts (a) through (c) below.

a. $ab(\frac{1}{a}+\frac{1}{b})$ $\qquad$ b. $cd(\frac{3}{c}+\frac{2c}{d})$ $\qquad$ c. $x(1-\frac{1}{x})$

d. What expression would go in the box in order to make the equation $\square\left(\frac{5}{x}+\frac{8}{y}\right)=5y+8x$ true?

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CL 6-121. Quinten and his sister Kelsey always make a habit of undoing each other's work. If Kelsey folds the laundry, Quinten unfolds it. If Quinten rakes the leaves in the yard, Kelsey "unrakes" them! While working on her math homework, Kelsey wrote the following equations. Help Quinten undo these equations by finding their inverse equations.

a. $y = 3x - 2$ b. $y = \frac{x+1}{4}$

c. $y = x^3 + 1$ d. $y = 1 + \sqrt{x+5}$

CL 6-122. Given the function $f(x) = 2 + \sqrt{x-1}$:

a. Graph $f(x)$ and state the domain and range.

b. Determine the equation for $f^{-1}(x)$, that is, the inverse of $f(x)$.

c. Graph $f^{-1}(x)$ using the appropriate new domain and range.

d. Compute $f^{-1}(f(5))$ and $f(f^{-1}(5))$ to show that your answer is correct.

CL 6-123. Use the definition of logarithms to compute each of the following *without a calculator*.

a. $\log_8(64) = x$ b. $\log_9(x) = \frac{1}{2}$

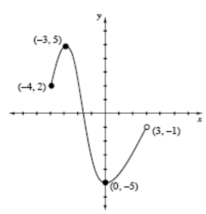
c. $\log_3(3^4) = x$ d. $10^{\log_{10}(4)} = x$

e. What do the answers to (c) and (d) demonstrate about logs and exponents with the same base?

CL 6-124. Use your Parent Graph Tool Kit or make a table to graph $y = \log_2(x)$.

CL 6-125. Use your answer to the previous problem to graph $y = 1 + \log_2(x - 3)$. State the equation of the new asymptote and the new x-intercept.

CL 6-126. Use the graph at right to help answer the questions below.



a. State the domain and range of the graph. Is this graph a function?

b. Draw the inverse of the graph. Is the inverse a function? Explain your answer.

c. State the domain and range of the inverse.

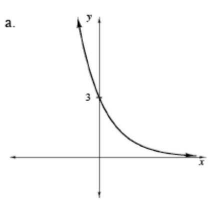
CL 6-127. A gallon of milk costs \$3.89. Inflation has steadily increased 4% per year.

a. What did a gallon of milk cost ten years ago?

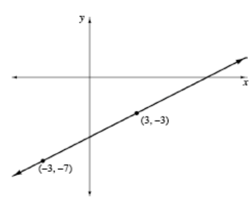
b. How much longer will it be until it costs \$10?

CL 6-128. Write possible equations for the graphs shown below.

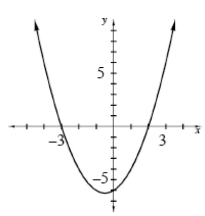
a.



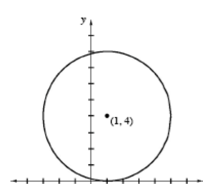
b.



c.



d.



CL 6-129. Factor the expressions in parts (a) through (d) below.

a. $3x^2 + 11x + 10$ b. $6x^3 - 31x^2 + 5x$

c. $6ab^2 + 15ab - 21a$ d. $y^2 + 5y - 24$

CL 6-130. Check your answers using the table at the end of this section. Which problems do you feel confident about? Which problems were hard? Have you worked on problems like these in previous math classes? Use the table to make a list of topics you need to learn more about, and a list of topics you just need to practice more.

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Functions and Their Inverses

Make up your own function and determine its inverse. Use colors and arrows to show connections between the multiple representations of a function and its inverse.

My Function:

Table of Function:

This is how I find the equation of the inverse:

Graphs of My Function and its Inverse

Its Inverse:

Table of Inverse:

The Big Ideas

- What is an inverse? What does it do? Why is it important?
- In what cases is the inverse a function?
- How are a function and its inverse related?
- How are the domains and ranges of a function and its inverse related?